Control of Mechanical engineering systems

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Mechanical and Aerospace Engineering

Everything should be made as simple as possible, but not simpler.
- Albert Einstein

Problem set 1
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Problem 1 (Suspension)**.**

Suppose you are an engineer working for an automobile company. You are asked to develop a simple mathematical model of the suspension system of a car. A simplified suspension system is shown in Figure 1. m_b represents the mass of the car body and the m_s that of the suspension system together with the axle and the wheel. The wheel acts as a spring (with spring constant k_w) that lies between the road surface and the axle. k_s is the spring constant of the suspension spring. $f(t)$ is an external force on the wheel due to road surface roughness etc.

Define y_1 and y_2 as the *deviation*s of the masses from their respective *equilibrium* positions. Derive a mathematical model (a system of differential equations) relating $y_1(t)$ and $y_2(t)$ to the forcing signal $f(t)$ in the two-mass system shown below. [Reading example 2.2 from the textbook may help]

Note: you should learn from this exercise that a mathematical model need not be a single *differential equation, but can be a number of differential equations that are* coupled.

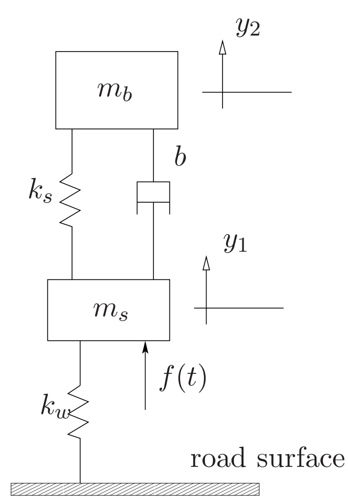

Figure 1: A simplified model of a suspension system of a car.

Problem 2 (Active suspension)**.**

You have been promoted to be a *R&D engineer* of the said automobile company (or demoted, according to your career goals). The company is considering building an active suspension system. One way to do so is to build shock absorbers with a changeable damping, where the damping coefficient is now a function of time: $b(t)$, which can be changed on the fly. Another possibility is to put an actuator in parallel with the suspension spring that can be commanded to produce a force $\eta(t)$ in opposite directions on the wheel axle and car body.

1. You are asked to model the active suspension when both of these methods are use. Modify the equations you derived in Problem 1 to include such inputs.
2. Is the resulting system linear?
3. Is it possible to use the force $\eta(t)$ to completely replace the spring and the shock absorber? Is it a good idea?

Problem 3.

1. Is the differential equation

$$\frac{d^3y}{dt^3}(t) + 5\frac{d^2y}{dt^2} + 2\dot{y}(t)u(t) + cos(2\pi y(t)) = u(t)$$

a linear differential equation?

2. Is

$$\frac{d^3y}{dt^3}(t) + 5\frac{d^2y}{dt^2} + 2\dot{y}(t) + y(t) = u(t)^2$$

a linear ODE (ordinary differential equation)?

3. Is

$$\frac{d^2y}{dt^2} + e^{-0.5t}\sin(\omega t)\dot{y}(t) + y(t) = u(t)$$

a linear ODE?

4. Does the ODE in Problem 3.3 represent an LTI system?

Problem 4 (Flow through an orifice + linearization)**.**

1. Read example Ex 2.14 and 2.16 from the textbook.
2. Starting from equations 2.63 and 2.67, re-derive the differential equation 2.71 yourself without looking at the textbook.