Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 10 / **Mini project** 2
Issued: April 13, 2009
Due: April 20, 2009 (5 pm)
Do all computations by hand unless stated otherwise.

Problem 1. Consider the following system of coupled differential equations:

$$\begin{aligned}\dot{X} &= -\sigma X + \sigma Y \\ \dot{Y} &= -XZ + rX - Y \\ \dot{Z} &= XY - bZ\end{aligned}$$

where σ, r, and b are constants. Is this a linear system? Why? [1]

Problem 2. Express the system with the following transfer function in the state space form:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{s^2 + 5s + 26}$$

Compare the poles of $G(s)$ to the eigenvalues of the $\mathbf{A}$ matrix.

Problem 3. In the following system, for what values of k and b is $(\mathbf{A}, \mathbf{B})$ controllable?

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -b & -k \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Problem 4. Consider the system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$ where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -0.98 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 10.78 & 0 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0.1 \\ 0 \\ -0.1 \end{bmatrix}$$

Is $(\mathbf{A}, \mathbf{B})$ completely controllable? (Hint: if the rank of the controllability matrix is not obvious to you, you can use the `rank` command in MATLAB to compute its rank)

[1] This system is the first known example of a system exhibiting "chaos", and is called the Lorentz attractor. Edward Lorentz (1917-2008), a meteorologist and mathematician derived these equations as a simple model of convention in a fluid medium (as part of an attempt to model the atmosphere). X is proportional to the intensity of the convective motion, Y is proportional to the temperature difference between the ascending and descending current, and Z is proportional to the deviation of the vertical temperature prole from linearity.

Problem 5. Determine whether the following systems are completely reconstructible.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -b & -k\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u \qquad \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -b & -k\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$y = [1 \quad 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix} \qquad y = [0 \quad 1]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Problem 6. Compute the transfer function from $U(s)$ to $Y(s)$ for the following system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -2 & -1 & 0\\ 0 & 0 & a\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}u$$

$$y = x_1$$

Problem 7. Consider the following model of a car, where $v(t)$ is the speed and $\theta(t)$ is the throttle angle

$$\dot{v} = -av + b\theta,$$

where a, b are positive constants. Now express the model in state space form with the states as position and velocity, assuming the measured output is the velocity. Is $(\mathbf{A}, \mathbf{C})$ completely reconstructible? Interpret your answer.

Problem 8. In the following sytem, is $(\mathbf{A}, \mathbf{C})$ completely reconstructible?

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ -2 & -1 & 0\\ 0 & 0 & a\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 1\\ 0\end{bmatrix}u, \qquad y = [1\ 0\ 0]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

Problem 9.

1. Show that the poles of the transfer function from $U(s)$ to $Y(s)$ in the following system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -0.02 & -0.2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u$$

$$\mathbf{y} = \begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

are at $-0.1 \pm 0.1j$. (Hint: you only need to compute the eigenvalues of A.)

2. Is it possible to design a state feedback controller for the this system so that the closed loop eigenvalues are at $-1 \pm j$? If so, design it. If not, explain why.

Problem 10. Consider the electromagnetic suspension system shown in Figure 1. The equation relating the position $h(t)$ of the suspended mass to the current $i(t)$ flowing through the electromagnet is:

$$m\ddot{h} = -F(i(t), h(t)) + mg + w(t), \tag{1}$$

where $w(t)$ is an external disturbance and $F(i(t), h(t))$ is the electromagnetic force:

$$F(i, h) = \frac{\mu_0 N^2 A}{4}\left(\frac{i(t)}{h(t)}\right)^2.$$

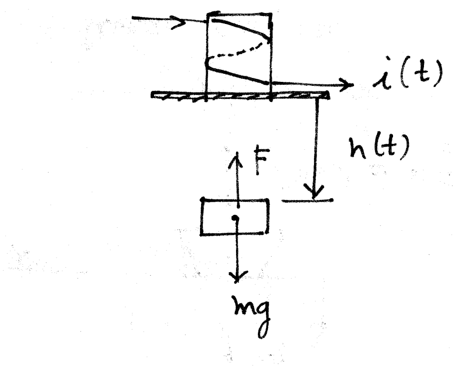

Figure 1: Electromagnetic suspension system

1. Determine the value of the mean current (call it i_0) so that $mg = F(i_0, h_0)$, where h_0 is the desired gap. If there were no disturbances ($w(t) \equiv 0$) and $h(\bar{t}) = h_0$, $\dot{h}(\bar{t}) = 0$ at some time $\bar{t}$, what will be the value of $h(t)$ for $t \geq \bar{t}$ if $i(t) = i_0$ for $t \geq \bar{t}$?

2. Now define $\tilde{i}(t) := i(t) - i_0$ and find a state-space representation of the system described by Eq. (1), with $x_1 = h, x_2 = \dot{h}$ as the states and $\tilde{i}(t), w(t)$ as the external inputs. Is this is a linear model?

3. Determine an equilibrium point of the system.

4. Linearize this system around the equilibrium point that corresponds to $\mathbf{x} = [h_0, 0]^T$, by defining $\tilde{x}_1 = x_1 - h_0, \tilde{x}_2 = x_2 - 0$.

5. Define the linear state space model you derived above in MATLAB using the `ss` command, for the following values of the parameters: $m = 750kg, A = 0.021m^2, N = 324, \mu_0 = 4\pi \times 10^{-7}$, and $h_0 := 0.008\ m$ is the *desired gap*[2]. Compute the trajectory $\tilde{\mathbf{x}}(t)$ of this linear system from $t = 0$ to $t = 10$ sec, when the current is held at i_0 and the disturbance w is zero, with the following initial condition: $\tilde{\mathbf{x}}(0) = [0.001mm, 0mm/s]^T$. Use the `lsim` command in MATLAB to compute the trajectory. Plot the states $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$ as a function of time (between 0 and 10 seconds).

Problem 11. Design an observer to estimate the state for the system with the following transfer function from the input $U(s)$ to output $Y(s)$:

$$\frac{Y(s)}{U(s)} = \frac{1}{s^2 + 2s}$$

(use y and $\dot{y}$ as the two states of the system when you convert it to state space form). In particular, choose the observer gain $\mathbf{L}$ so that the eigenvalues of $\mathbf{A} - \mathbf{LC}$ are at $-3 \pm 4j$.

[2] There correspond to the values for an experimental Maglev train, and are taken from the paper "Fault-tolerant Control for Maglev Suspension System Based on Simultaneous Stabilization", by Zhizhou Zhang, Zhiqiang Long, Longhua She and Wensen Chang, published in the Proceedings of the IEEE International Conference on Automation and Logistics, August 18 - 21, 2007, Jinan, China.