Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 2
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Problem 1.

1. Show (by using the definition of Laplace transform) that $e^{pt} \xrightarrow{\mathcal{L}} \frac{1}{s-p}$, with ROC specified by $Re(s) > Re(p)$, where p is a complex number.
2. Repeat the problem above after dinner.
3. Repeat the problem above before breakfast.
4. Set the alarm at 2 a.m., get up, and redo the problem.

Problem 2. Prove the following (by using the definition of Laplace transform)

1. $\delta(t) \xrightarrow{\mathcal{L}} 1$.
2. $1(t) \xrightarrow{\mathcal{L}} \frac{1}{s}$.
3. $\sin(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{\omega_0}{s^2+\omega_0^2}$.
4. $\cos(\omega_0 t) \xrightarrow{\mathcal{L}} \frac{s}{s^2+\omega_0^2}$.
5. If $x(t) \xrightarrow{\mathcal{L}} X(s)$ and $y(t) \xrightarrow{\mathcal{L}} Y(s)$, then $\alpha x(t) + \beta y(t) \xrightarrow{\mathcal{L}} \alpha X(s) + \beta Y(s)$, where α and β are scalars (real or complex).
6. If $y(t) \xrightarrow{\mathcal{L}} Y(s)$, and $z(t) \triangleq \dot{y}(t)$, then $z(t) \xrightarrow{\mathcal{L}} sY(s) - y(0)$. (hint: use integration by parts)

Problem 3. Prove the following (use the results of the previous problem)

1. If $y(t) \xrightarrow{\mathcal{L}} Y(s)$, and $z(t) \triangleq \int_0^t y(\tau)d\tau$, then $z(t) \xrightarrow{\mathcal{L}} \frac{1}{s}Y(s)$.
2. $t(t) \xrightarrow{\mathcal{L}} \frac{1}{s^2}$.
3. $te^{pt} \xrightarrow{\mathcal{L}} \frac{1}{(s-p)^2}$