Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 3
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Problem 1. Find the signals (that is, the time functions) corresponding to the following Laplace transforms by using partial fraction expansion. To find the inverse transforms, use the properties of Laplace transforms discussed in the class, and the facts that (i) $e^{pt} \xrightarrow{\mathcal{L}} \frac{1}{s-p}$ for any scalar p (real or complex), and (ii) $\delta(t) \xrightarrow{\mathcal{L}} 1$. Do all calculations by hand.

1. $X(s) = \dfrac{2}{s(s+2)}$.
2. $X(s) = \dfrac{3s^2+9s+12}{(s+2)(s^2+5s+11)}$.
3. $X(s) = \dfrac{3s^2+9s+12}{s^2-6s+9}$.

Problem 2. Find the signals (that is, the time functions) corresponding to the following Laplace transforms. Use the result stated in Problem 3.3 of HW#2.

1. $X(s) = \dfrac{2(s^2+s+1)}{s^3+2s^2+s}$.
2. $X(s) = \dfrac{2(s+2)(s+5)^2}{(s+1)(s^2+4)^2}$.

Problem 3.

1. Fill in the blanks of the following statement: " If $g(t_1) \triangleq \int_0^{t_1} y(\tau)1(t_1-\tau)d\tau$, then $g(t_1) = y(-) * 1(-)$".
2. Given that the Laplace transform of $y(t)$ is $Y(s)$, find the Laplace transform of $f(t) \triangleq \int_0^t g(t_1)dt_1$, where g is defined above. (hint: use (1) the first problem, and (2) convolution property of Laplace transforms.)

Problem 4. Solve the following differential equations using Laplace transforms. Note that the solution to the second and the third equations will be complex functions of time. The calculations required to solve the fourth can be reduced by using the solution to the second and the third and using superposition property for linear differential equations.

1. $\ddot{y} + \dot{y} + 3y = 0; y(0) = 1, \dot{y}(0) = 2$.
2. $\ddot{y} + \dot{y} + 3y = e^{5jt}; y(0) = 1, \dot{y}(0) = 2$.
3. $\ddot{y} + \dot{y} + 3y = e^{-5jt}; y(0) = 1, \dot{y}(0) = 2$.

4. $\ddot{y} + \dot{y} + 3y = \sin(5t); y(0) = 1, \dot{y}(0) = 2.$

Problem 5. Compute the poles and zeros of the following transfer functions (by hand):

1. $H(s) = \dfrac{1}{s^2 + 36}$
2. $H(s) = \dfrac{2s + 3}{s^2 + 2s + 36}$
3. $H(s) = \dfrac{2s^2 + s - 6}{s^3 + 2s^2 + 12s}$

Problem 6. Compute the poles and zeros of the following transfer functions using MATLAB (use the `zpkdata` command):

1. $H(s) = \dfrac{1}{s^2 + 24.525}$
2. $H(s) = \dfrac{1}{s^2 + 9.4055s + 24.525}$
3. $H(s) = \dfrac{0.5s^3 - 0.2s^2 + 0.445s - 0.3030}{s^5 + 15.9s^4 + 692.05s^3 + 4460.4598s^2 + 11171.725s + 11011.5252}$

The first two are the transfer functions from disturbance to the normalized deviation in the feedback controlled Maglev system discussed early in the course. The first one is with P-control ($K_P = 26.3358$) and second one is with PD control ($K_P = 26.3358, K_D = 0.1$).