Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Scientific questions often have a surface appearance of dumbness .... They are asked in order to prevent dumb mistakes later on.

- Robert Pirsig, in *Zen and the art of motorcycle maintenance*

Problem set 4
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Problem 1. What is the characteristic polynomial of the following transfer function? What is the characteristic equation? What are the zeros and poles of the transfer function?

$$H(s) = \frac{s^2 + 5s + 45}{s^3 + 2.4s^2 + 5s + 5000000}$$

Note: Find the poles and zeros using the `roots` command in MATLAB. The roots of the equation $x^4 + 3x^3 - 5x^2 + 2x + 10 = 0$ can be computed in MATLAB by typing `roots([1, 3, −5, 2, 10])` at the MATLAB command prompt. Similarly, to find the roots of $4x^3 + x + 67 = 0$, type `roots([4, 0, 1, 67])`.

Problem 2. Answer the following questions without doing any calculations:

1. Suppose the transfer function $H(s)$ from input to output of a system has the following characteristic polynomial:

   $$s^6 + 2.33s^5 + 23s^3 + 456.44s^2 + 34597s + 2.3 \times 10^4$$

   Is the plant BIBO stable?

2. Is a plant with the following characteristic polynomial BIBO stable?

   $$s^6 + 2.33s^5 + 4s^4 + 23s^3 + 456.44s^2 + 34597s$$

Problem 3. Consider the second order polynomial equation $x^2 + bx + c = 0$. We know from the first special case of the Routh stability criteria that it is necessary for b and c to be positive for the roots to be on the LHP. Prove that for a second order system, it is also sufficient. That is, prove that if b and c are positive, then the roots of the equation are in the LHP.

Problem 4. In the University of Nastyville, there is professor (Dr. Monstrous), who gives his students a 10 minute test every day, to ensure that they keep up with the material covered in the class. After considerable modeling effort, it was determined that the transfer function from $u(t)$ and $y(t)$, where

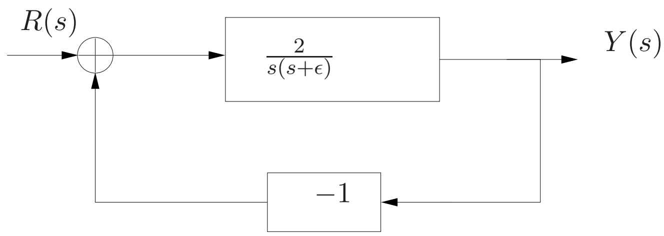

Figure 1:

$u(t)$ is the number of hours a student studies (t is measured in days) and $y(t)$ is her score on the test on the t-th day, is the following

$$H(s) = \frac{50a_2}{s^2 + a_1 s + a_2}$$

where a_1 and a_2 are fixed constants, but whose values are only approximately known. Specifically, it is known that $a_1 = 1 \pm 0.1$ and $a_2 = 2 \pm 0.9$. What conclusion can you draw about the stability of this transfer function? Give reasons for your answer.

Problem 5. In the system shown in Figure 1, ϵ is a non-negative constant.

1. Find the closed loop transfer function from $R(s)$ to $Y(s)$, and an expression for its poles. What are the poles when $\epsilon = 2$? Sketch these poles on the complex plane.

2. Let $r(t) = 5 \times 1(t)$. Assuming $\epsilon > 0$, what is the steady state value of $y(t)$? Can you use the Final Value Theorem to determine the steady state values of $y(t)$ if $\epsilon = 0$? If $\epsilon = 0$, is there a steady state value of $y(t)$? Give reasons for your answers.

3. Let $\epsilon = 2$. Use the `lsim` command in MATLAB to numerically determine the output $y(t)$ for the input $r(t) = 5 \times 1(t)$ from $t = 0$ to $t = 20$ seconds, assuming all initial conditions are zero. Plot $u(t)$ and $y(t)$. (See the end of this homework for an example of a MATLAB script to do such a simulation.)

Problem 6. Consider the following system:

$$\ddot{y} + 0.2\dot{y} - 5y = u(t) + w(t)$$

where $u(t)$ is a control input and $w(t)$ is a disturbance.

1. Find the *plant transfer function* $P(s)$ from $V(s)$ to $Y(s)$, where $V(s) \triangleq W(s) + U(s)$. Draw a block diagram showing the input-output relationships between $W(s)$, $U(s)$ and $Y(s)$.

2. Now suppose we want $y(t)$ to track a reference signal $r(t)$ by using feedback control. Let us define the *error* signal $e(t) \triangleq r(t) - y(t)$. We will use a feedback controller that will produce an appropriate control signal $u(t)$ based on the error $e(t)$, with the goal of making $e(t) \to \infty$ as $t \to \infty$ when there are no disturbances. Draw a block diagram showing the feedback loop, with $C(s)$ as the controller transfer function, which is to designed.

(a) If you want to use a proportional controller ($u(t) = K_P\, e(t)$) so that the closed loop transfer function from $R(s)$ to $Y(s)$ is BIBO stable, what is the minimum value of K_p that is allowed? Choose a K_p so that the closed loop is BIBO stable, and plot the resulting closed loop poles. Can you modify the real parts of the closed loop poles by changing K_p?

(b) When $r(t) = r_0 1(t)$, where r_0 is a constant, does the error $e(t)$ go to zero as $t \to \infty$ when there are no disturbances? Why or why not?

(c) Design a controller to make the *steady state tracking error* zero (ensure closed loop stability first) when there are no disturbances.

(d) Simulate the closed loop system with the controller designed in part (c), for the following signals: $r(t) = 2 \times 1(t)$ and $w(t) = 0.1\sin(5t) + 0.1\sin(10t + 30^0)$. Plot $r(t)$, $y(t)$, and $u(t)$.

Note on simulating LTI systems: Suppose we want to compute numerically in MATLAB the output of a plant to a step input, whose transfer function from input to output is:

$$H(s) = \frac{Y(s)}{U(s)} = \frac{4(s+1)}{s^3 + 4s^2 + 6s + 4}. \tag{1}$$

A polynomial is specified in MATLAB by its coefficients. So the numerator is specified as [4, 4] and the denominator is specified as [1, 4, 6, 4]. You can define a transfer function by the `tf` command. To define the transfer function shown in equation (1), type the following at the MATLAB command prompt:

```
>> num = [4 4]
>> den = [1  4  6  4]
>> H = tf(num,den)
```

To simulate the system, we have to define a vector of time instants at which we want to determine the value of y, and the values of the input signal at those time instants. Say we want to compute $y(t)$ for $t = 0$ to 10 seconds at 1 millisecond intervals, for the unit step function as the input, with zero initial conditions. This can be done by typing the following at the MATLAB command prompt:

```
>> Ts = 0.001;
>> t = [0:Ts:10];
>> u = ones(1,length(t));
>> y = lsim(H,u,t);
```

The result can be plotted by creating a figure window and plotting the vector y:

```
>> figure;
>> plot(t,u,'r',t,y,'b');
>> legend('input','output');
```