Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 5
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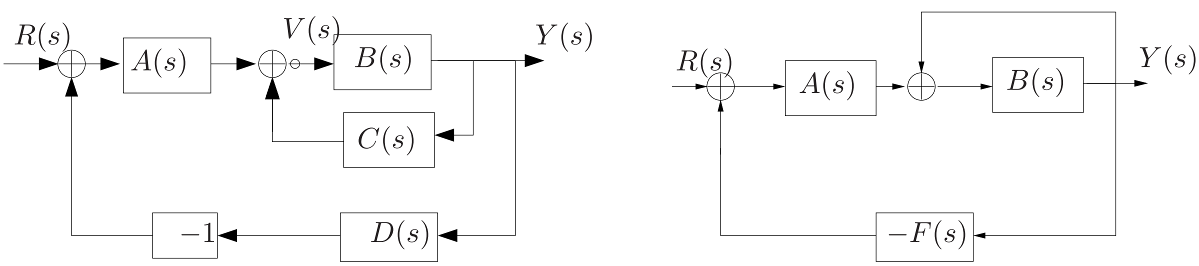
Problem 1. Find the following transfer functions:

1. The transfer function from $R(s)$ to $Y(s)$ in the block diagram shown in Figure 1(a).
2. The transfer function from $V(s)$ to $Y(s)$ in the block diagram shown in Figure 1(a).
3. The transfer function from $R(s)$ to $Y(s)$ in the block diagram shown in Figure 1(b).



(a) (b)

Figure 1:

Problem 2. Find the transfer function from $R(s)$ to $Y(s)$ in the block diagram shown in Figure 2. (Hint: this cannot be answered by applying the formula "forward gain by one minus the loop gain" since there are multiple intersecting feedback loops. The trick is to call the signal going into B, say, $V(s)$, and then finding the transfer function from R to V by writing the relationships among R, V, and Y. The desired transfer function is simply B times the transfer function from V to Y.)

Problem 3. The relationship between a satellite's attitude θ, the control force u (from reaction jets) and disturbance torque w (due to solar radiation pressure), can be modeled as

$$I\ddot{\theta}(t) = du(t) + w(t),$$

where I is the satellite's moment of inertia, and d is the moment arm between the reaction jets' force and the satellite's center of mass.

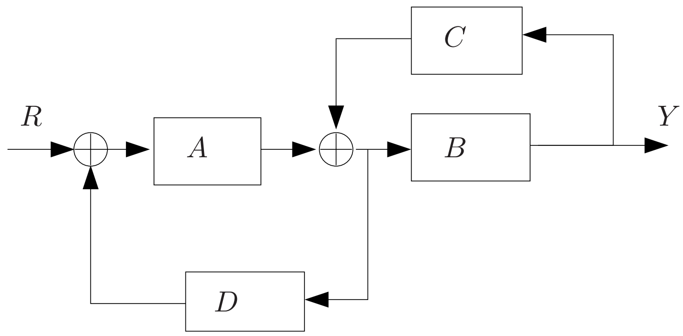


Figure 2:

1. What is the transfer function from $U(s)$ to $\Theta(s)$? Express the transfer function in the form such that the highest power of s in the denominator has 1 as its coefficient[1]. Draw a block diagram to show the relationships among $U(s)$, $W(s)$ and $\Theta(s)$.

2. Suppose we want to control the system so that the measured signal $\theta(t)$ tracks some reference signal $r(t)$ closely. Define the *tracking error* $e(t) \triangleq r(t) - \theta(t)$, let $C(s)$ be the controller transfer function from $E(s)$ to $U(s)$. Draw a block diagram of the feedback control loop with the controller transfer function.

3. What is the (closed-loop) transfer function from $R(s)$ to $\Theta(s)$? From $W(s)$ to $\Theta(s)$? From $W(s)$ to $U(s)$?

4. The aspect of designing the controller $C(s)$ is to ensure that all possible closed loop transfer functions are BIBO stable. When that is the case, we say the *closed loop is stable*. Suppose someone designs a controller $C(s)$ and asks you to check his calculations to make sure that the closed loop is stable. How many characteristic equations do you have to solve to make sure the poles are on the LHP? Why?

Problem 4. In the problem above,

1. Is it possible to design a Proportional (P) controller so that the closed loop is BIBO stable? If possible, design one, otherwise, provide reasons.

2. Is it possible to design a Proportional-Derivative (PD) controller so that the closed loop is BIBO stable? If possible, design one, otherwise, provide reasons.

3. Is it possible to design a Proportional-Integral (PI) controller[2] so that the closed loop is BIBO stable? If possible, design one, otherwise, provide reasons.

4. Design a Proportional-Integral-Derivative (PID) controller so that the closed loop is BIBO stable. Is the steady state error (when there are no disturbances) in response to a step input as a reference, is zero?

[1] Such a polynomial is called a *monic* polynomial

[2] A PI control law has the only the proportional and integral terms: $u(t) = K_P e + \int_0^t e(\tau)d\tau$.

5. Is it possible to design a PID controller so that in addition to closed loop stability, the steady state error (when there are no disturbances) for a sinusoidal reference signal $r(t) = 0.1\sin(\omega_0 t)$ is zero?

Problem 5. Consider the following transfer function from $U(s)$ to $Y(s)$:

$$G(s) = \frac{s}{s^2 + 2s + 10}$$

1. Can you apply the Final Value Theorem to find the steady state value of $y(t)$ when $u(t) = 1(t)$? If yes, find the steady state value. Otherwise, give reasons.
2. Can you apply the Final Value Theorem to find the steady state value of $y(t)$ when $u(t) = t$? If yes, find the steady state value. Otherwise, give reasons.
3. Can you apply the Final Value Theorem to find the steady state value of $y(t)$ when $u(t) = 5\sin(40\pi\ t)$? If yes, find the steady state value. Otherwise, give reasons.

Problem 6. Draw a Bode plot of $G(s) = \frac{50}{s+50}$ (1) approximately by hand and (2) using the `bode` command in MATLAB. Answer the following questions by using the bode plot produced by MATLAB.

1. If this system is driven by an input signal $u(t) = 2\sin(5t + 30^0)$, what is the output $y(t)$ at steady state?
2. If this system is driven by input signal $u(t) = \cos(2t)$, what is the output $y(t)$ at steady state?
3. If this system is driven by a sinusoidal signal with a frequency of 8 Hz, at steady state what is the amplitude of the output sinusoid? (don't forget to convert to radian/sec). What is the phase difference between the input and the output at steady state? That is, what is (phase of output - phase of input)?
4. If this system is driven by a sinusoidal signal with a frequency of 80 Hz, at steady state what is the amplitude of the output sinusoid?
5. Now answer these questions from the hand-drawn Bode plot. Which answers have large errors? Why?