Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 6

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Problem 1. *Bode plot of systems with real poles and zeros*

Sketch the Bode plot of the following transfer functions. Be specific about the x and y-scales. For each problem, verify your result using MATLAB.

1. $G(s) = \dfrac{1}{s}$
2. $G(s) = \dfrac{1}{s^2}$
3. $G(s) = s$
4. $G(s) = s^2$
5. $G(s) = \dfrac{1}{(s+2)(s+20)}$
6. $G(s) = \dfrac{s+5}{(s+0.01)(s+100)}$

Problem 2. *Bode plot of systems with complex poles and zeros*

Sketch the Bode plot of the following transfer functions. Be specific about the x and y-scales. For each problem, verify your result using MATLAB.

1. $G(s) = \dfrac{1}{s^2+3s+10}$
2. $G(s) = \dfrac{s}{s^2+3s+10}$
3. $G(s) = \dfrac{1}{s(s^2+3s+10)}$
4. $G(s) = \dfrac{s^2+2s+8}{s(s^2+2s+10)}$

Problem 3. *Effect of zeros on the right half plane* Sketch and compare the Bode plots of the following two transfer functions. Plot using MATLAB to verify your sketches:

$$G_1(s) = \frac{s+2}{s^2+8s+12} \qquad G_2(s) = \frac{s-2}{s^2+8s+12}$$

You'll see that only the phase curves differ between these two systems, their magnitude curves are the same.

Problem 4. Consider

$$G_1(s) = \frac{1}{s^2 + 8s + 12} \qquad G_2(s) = \frac{-1}{s^2 + 8s + 12}$$

Answer the following questions without drawing the Bode plots:

1. Is it true that the magnitude curves in the Bode plots of these systems are the same? Why?
2. For $G_1(s)$, what are low frequency and high frequency limits of the phase curve of its Bode plot? Meaning, what are the values of $\angle G_1(j\omega)$ as $w \to 0$ and $\angle G_1(j\omega)$ as $w \to \infty$?
3. Answer the question above for $G_2(s)$.

Problem 5. Suppose a sine sweep experiment has been conducted on a system $G(s)$ and its frequency response has been experimentally determined, which is shown in the Figure 1[1]. From the plot, answer the following questions:

1. If it is known that $G(s)$ does not have any zeros, what is the order of the denominator of $G(s)$? What is the relative degree of $G(s)$?
2. Are all the poles of $G(s)$ real?
3. The experimentally obtained values of $G(j\omega)$ for 100 values of the frequency ω are available in the file `sine_sweep_data.mat`. The file contains three arrays `W`, `M_data`, and `P_data`, which contain frequencies (rad/sec), and the magnitude and phase (degree) of the system at those frequencies, respectively. Note that the magnitudes in `M_data` are in absolute values, not in dB. Use the `invfreqs` command in MATLAB to fit a transfer function $\hat{G}(s)$ to this data. Then compare the bode plot of $\hat{G}(s)$ with the experimental data to see how good the fit is.

Problem 6. If $G(s)$ is stable, prove that when the input to $G(s)$ is $r_0 1(t)$, the steady state output is " D.C. gain of $G(s)$ times r_0". (Remember: the D.C. gain of $G(s)$ is $G(0)$).

Problem 7. What is the natural frequency and the damping ratio of $G(s) = \dfrac{s-3}{5s^2 + 2s + 1000}$? What is its DC gain? If the input to the system is $1(t)$, what is the steady state output?

Problem 8. Let $G(s) = \frac{2}{s^2+0.2s+7}$ be the transfer function of a plant.

1. What are the rise time, settling time, and peak overshoot of this plant (in response to a unit step input)?
2. Plot the step response of $G(s)$ using the `step` command in MATLAB.

Problem 9.

1. Problem 3.23 from the Textbook.
2. Problem 3.24 from the Textbook.
3. Problem 3.26 from the Textbook.

[1] Note that experimentally you can determine the gains and phases at only a finite number of frequencies, hence it is not a continuous curve

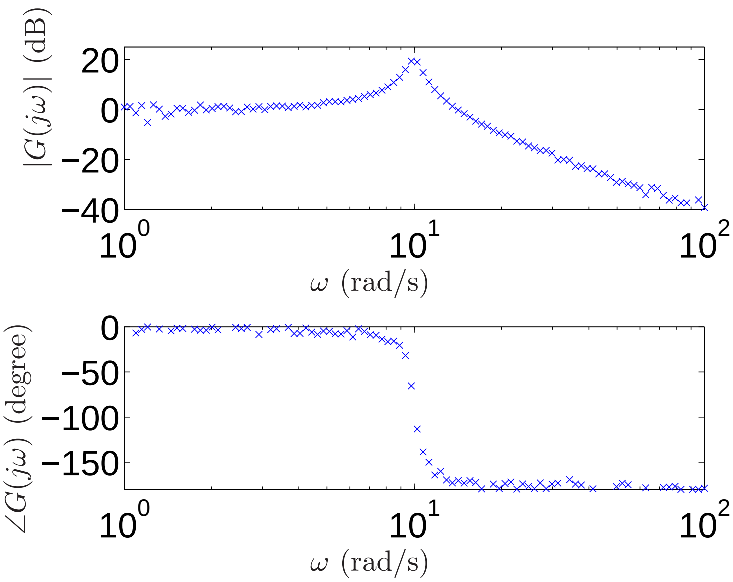
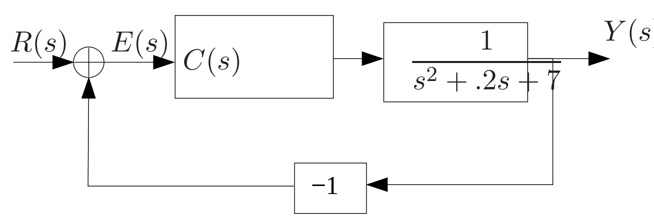


Figure 1: Experimentally obtained frequency response of $G(s)$ (Problem 5).

Problem 10. Suppose the step response of $G(s) = \frac{2}{s^2+0.2s+7}$ is deemed too slow, and we wish to improve the step response of the plant by using close loop control, as shown in the figure. Specifically, we wish to design a controller to meet the following specifications for the closed loop's step response: (i) $t_r \leq 0.2$ sec, (ii) $t_s \leq 10$ sec, (iii) $M_p \leq 0.1$. As a first attempt, we try to meet these specifications by using a proportional controller, that is $C(s) = k_p$, where the gain k_p is to be chosen.



1. Write the closed loop transfer function from $R(s)$ to $Y(s)$ and find the expressions for the damping ratio and natural frequency of this transfer function as a function of k_p.

2. Among the three parameters: rise time, settling time, and peak overshoot, which can be changed by varying k_p?

3. Determine the minimum value of the proportional gain k_p so that the rise time specification is met.

4. For this value of k_p, what are the values of the peak overshoot and the settling time? Have they improved over the corresponding values for the open loop plant or deteriorated?