Control of Mechanical engineering systems

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Mechanical and Aerospace Engineering

Problem set 7
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Problem 1 (Low pass filter design). It is noticed that the measurements of an aileron angle of an aircraft's wing is corrupted by high frequency noise. To reduce the noise level, a low pass filter $F(s)$ is to be designed, so that the sensor readings are to be passed through the filter before used in the control loop. The specifications are that the amplitude of every component of the input signal (to the filter) whose frequency is lower than 10 rad/sec should be untouched by the filter, whereas the amplitudes of all components whose frequencies are higher than 100 rad/sec should be reduced as much as possible.

1. These vague requirements are translated by two engineers into the following specifications. Which one makes more sense, and why?
   (a) $|F(j\omega)| \leq 1$ for all $\omega < 10rad/s$, and $|F(j\omega)| < 0.1$ for all $\omega > 500rad/s$.
   (b) $0.99 < |F(j\omega)| < 1.01$ for all $\omega < 10rad/s$, and $|F(j\omega)| < 0.1$ for all $\omega > 500rad/s$.
2. Verify, by plotting the Bode plots (use MATLAB) that both of the following two designs satisfy the specifications (b) above.
   (a) $F(s) = \dfrac{20000}{s^2 + 285.7s + 20000}$.
   (b) $F(s) = \dfrac{20000}{s^2 + 140s + 20000}$.

   However, one of them is a poor design. Which one, and why?
3. The file `noisy_data.mat` contains three arrays: `sensor_data` (the measured signal), `time` (in sec, the times at which the sensor data was sampled), and `true_signal` (the "true signal" being measured, though in practice you'll never have access to this). Filter the noisy sensor signal by each of the filters above and compare the outputs.

Problem 2. Design a second order low pass filter $F(s)$ to meet the following specifications: $0.988 < |F(j\omega)| < 1.000$ for $\omega < 10rad/s$, and $|F(j\omega)| < 0.2$ for $\omega > 250$ rad/s. [hint: you can find one by trial and error using MATLAB. Use a second order system, choose a ζ that is larger than one (to get real poles) and vary the natural frequency till the magnitude curve of the Bode plot tells you the specs are met.]

Problem 3. Problem 3.36 from the Textbook.

Problem 4. For $k = 3$ in the problem above,the closed loop transfer function from the pitch command θ_r to pitch angle θ is

$$G_{\theta_r,\theta} = \frac{150s^3 + 900s^2 + 1650s + 900}{s^5 + 15.03s^4 + 240.5s^3 + 1304s^2 + 1667s + 924}.$$

1. Find the poles and zeros of this transfer function (you can use `zpkdata` command in MATLAB).
2. First recall that for a complex conjugate pole pair $p, \bar{p}$, the damping ratio ζ and natural frequency ω_n of *that pole pair* is defined implicitly by the following equation:

$$(s - p)(s - \bar{p}) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

Now, compute the natural frequency and damping ratio of each of the pole pairs of $G_{\theta_r,\theta}$ specified above. Compute the step response of $G_{\theta_r,\theta}$ in MATLAB by using the `step` command, read the rise time, peak overshoot, and settling time of $G_{\theta_r,\theta}$ off the plot of the step response, and then compare these numbers to what you'd get if you apply the formulas $t_r \approx \frac{1.8}{w_n}$ etc to the natural frequencies and damping ratios of each of the pole pairs. Are these formulas useful in predicting rise time etc. of $G_{\theta_r,\theta}$? (Note: quantities such as rise time and peak overshoot can be measured in MATLAB by using the `stepinfo` command. You may use it get an independent verification.)

Problem 5 (disturbance rejection)**.** Consider the feedback loop for the speed control of a magnetic tape-drive system is shown in the Figure 1. Use $J = 0.10Kg \cdot m^2, b = 1.00N \cdot m \cdot sec$. Show that the system can track a step references with zero steady state error even when disturbances are present, if the disturbance is also a step signal. Is it true irrespective of what value of K is chosen?

(Hint: Express the speed $V(s)$ as the sum of two terms

$$V(s) = H_{rv}(s)V_r(s) + H_{dv}(s)D(s) \tag{1}$$

where $V_r(s)$ is the (Laplace transform of) the reference speed command. Thus, we have

$$y(t) = y_r(t) + y_d(t) \tag{2}$$

where $y_r(t) = \mathcal{L}^{-1}(H_{rv}(s)V_r(s))$ is the response due to the reference and $y_d(t) = \mathcal{L}^{-1}(H_{dv}(s)D(s))$ is the response due to the disturbance. Now show that when $r(t) = r_0\ 1(t)$ and $d(t) = d_0\ 1(t)$, we have $y_r(t) \to r_0$ and $y_d(t) \to 0$ as $t \to \infty$.)

Problem 6. A particular closed loop control system has the following specifications: $t_r \leq 0.0010$ sec, peak overshoot $M_p \leq 16\%$, and settling time $t_s \leq 0.52$ sec, and steady state error to a step input less than 0.12.

1. Sketch the allowable region in the complex plane where the dominant poles of the closed loop transfer function may lie to satisfy these requirements.
2. The control loop consists of a plant (with transfer function $P(s)$) in a unity gain feedback loop so that the closed loop transfer function from the reference $R(s)$ to output $Y(s)$ is $Y/R = P/(1+P)$. What condition should $P(j\omega)$ satisfy near $\omega = 0$ to meet the specifications, that is, what is the required low frequency asymptotic behavior of $P(s)$?

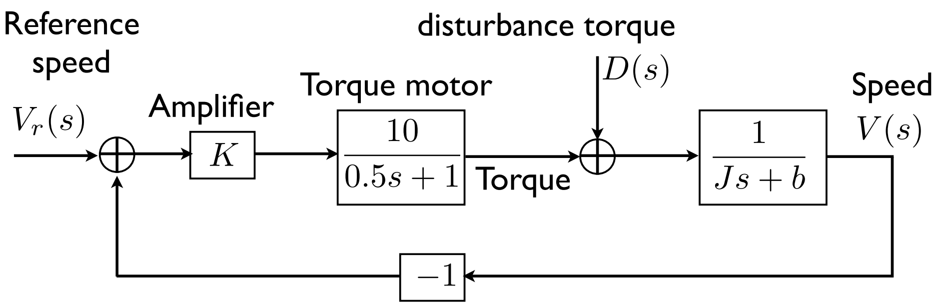

Figure 1: Magnetic tape drive control system.

Problem 7 (Sensitivity to parameter variations)**.**

1. Problem 4.3 from the text.
2. (K, H_1, and H_2 are positive real numbers.) Choose K so as to make the sensitivity to parameter variation less than 0.1 in the second design (Figure 4.32 (b)). Is it possible to achieve this in the first design (Figure 4.32 (a)) by choosing K?

Problem 8. Problem 5.1 of the text. For part (a) an (b), plot the root locus using MATLAB.