Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 8
Issued: March 25, 2009

Problem 1. 1. Express the following transfer function in a form suitable for plotting the root locus with respect to the parameter k:

$$H(s) = \frac{5ks + 10}{s^2 + (5k+2)s + 10}$$

2. Now, plot the real axis part of the root locus, and draw the asymptotes.

3. Plot the complete root locus using the `rlocus` command in MATLAB. Then, write a matlab script to compute the poles of

$$H(s) = \frac{5ks + 10}{s^2 + (5k+2)s + 10}$$

as k varies from 0 to 1000 and plot these poles on top of the root locus you plotted above. What did you learn?

Problem 2. : Consider the feedback loop shown in 1, where the plant is $P(s) = \frac{1}{s^2}$, the controller is $C(s) = k\frac{1000+400s}{s+1000}$ (the gain k is to be decided), and $F(s)$ is a (optional) low pass filter to reduce sensor noise. For the following three possible choices of $F(s)$, draw the real axis part and asymptotes of the root locus for the closed loop poles (with respect to k)

1. $F(s) = \frac{100}{s^2+10s+100}$
2. $F(s) = \frac{100}{s+100}$
3. $F(s) = 1$

Among these three choices of $F(s)$, which one ensures that the closed loop stays stable for arbitrarily large values of the control gain k? Now plot the complete root loci for these three choices using MATLAB.

Problem 3. Consider the feedback loop in Figure 1 with $F(s) = 1$. The plant $P(s) = \dfrac{1}{s^2 - 6}$ is open-loop unstable (remember Maglev?). It is stabilized by a PD controller $C(s) = k_p + s\ k_d$ (never mind that the derivative part of the PD controller is difficult to implement).

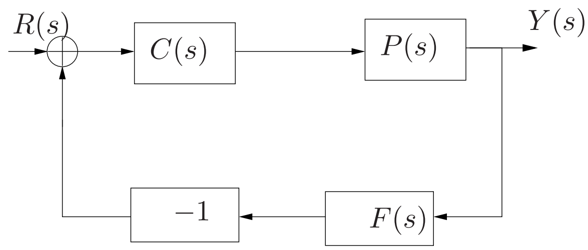


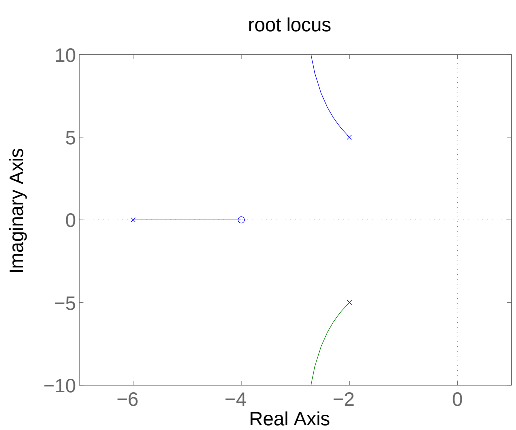
Figure 1: Feedback loop for Problem 2 and 3.

1. Choose $k_d = 2$ and sketch the root locus of the closed loop poles with respect to the gain k_p and verify your sketch using MATLAB. (You'll see that in this case you can sketch the entire root locus by using only the rules that you learned in class.)

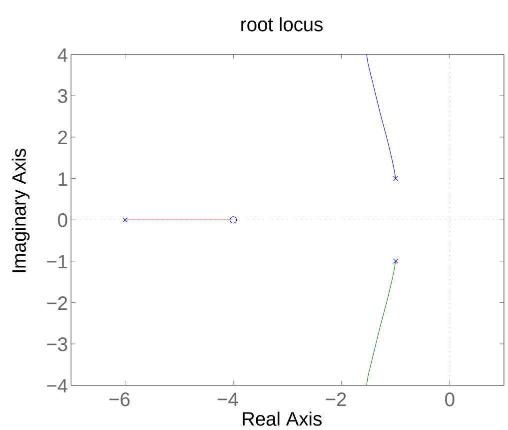
2. With k_d chosen as 2, determine a value of k_p that will ensure that the rise time of the closed loop transfer function from $R(s)$ to $Y(s)$ will be less than 1 sec. (Don't forget to take into account the fact that the closed loop transfer function has a zero)

Problem 4. 1. The root locus of L (that is, the roots of the equation $1 + kL(s)=0$ for various values of k) are shown in Figure 2(a). What is $L(s)$?

2. Let $H = \frac{CP}{1+kCP}$ be a closed loop transfer function, where $CP = \frac{s+4}{(s+6)(s^2+2s+2)}$. The root locus of the closed loop poles of H are shown in Figure 2(b). Is it possible for H to have a damping ratio greater than 0.4472 and a natural frequency greater than 3.6 rad/sec? If so, for what value of k will make it happen, and if not, why not? (Hint: Answer this question by drawing the allowable region in the complex plane where the poles of H must lie to satisfy the requirements (on top of the root locus provided) and then checking if it is possible for H to have poles in that region. A ruler, protractor, and a drawing compass will help. Note that for higher order systems, natural frequency and damping ratio are computed from the least stable complex pole pair. You can answer this question by not using the root locus at all and just applying formulas, but doing it using the root locus will help you in the test.)



(a) Problem 4(1)



(b) Problem 4(2)

Figure 2:

Problem 5.

The root locus of $H(s) = \frac{k+2s}{1+kL(s)}$, where $L(s)$ is second order, is shown in Figure 3. Can the settling time of $H(s)$ be made less than 2.3 seconds by choosing k appropriately?

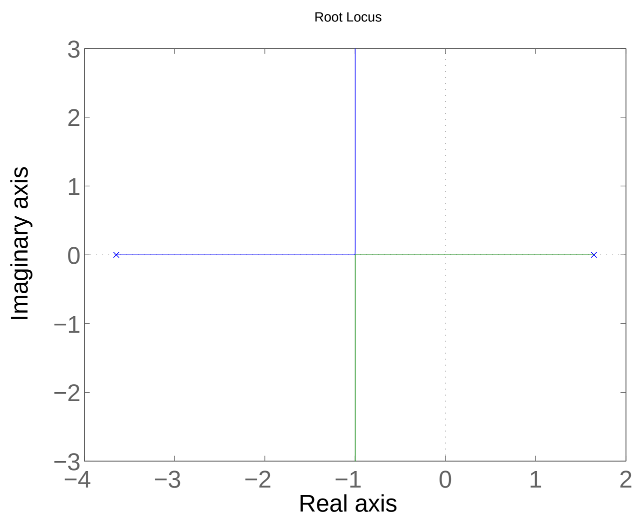


Figure 3: Root locus of $H(s)$

Problem 6.

1. Compute the 1st and 2nd order Pade approximations to $e^{-0.1s}$.
2. Compute the 2nd order and 3rd order Pade approximations to e^{-2s}.
3. Compute a 4-th order rational approximation, $\hat{G}(s)$, to the transfer function $G(s) = \frac{s+2}{s^2+3s+67}e^{-0.5s}$ (4-th order means the order of the denominator of $\hat{G}$ is 4).

Problem 7. Suppose the transfer function from the throttle angle to the speed of a car is $P(s) = \frac{5}{s+4}e^{-0.05s}$ (that is, a first order system with a delay of 0.05 seconds). Let a standard negative feedback loop be used to control this plant with a lead controller $C(s) = k\frac{s+5}{s+10}$, where k is a to-be-designed control gain. What will happen to the closed loop as $k \to \infty$? Now plot the root locus of the closed loop by taking a first order Pade approximation of the delay (use Matlab to plot the root locus). Does the root locus match your prediction?

Review problems/Suggested exercises

Problem 8. Sketch the real axis part of the root locus and its asymptotes for

$$L = \frac{s+6}{(s^2+4s+8)(s^2+1s+5)}.$$

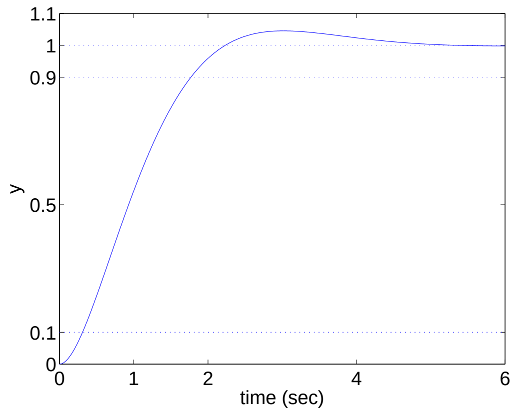


Figure 4: Step response of a plant

Problem 9. The output of a system in response to an unit step is shown in Figure 4. What is the rise time (in seconds) and peak overshoot of the system?

Problem 10. What are the natural frequency and damping ratio of the following systems?

$$G_1 = \frac{1}{0.75s^2 + 31s + 45}$$
$$G_2 = \frac{10}{2s^2 + 2s + 45}$$

what is the difference in their peak overshoots? relate the difference to the pole locations.

Problem 11. The transfer function from the elevator angle δ_e to the pitch attitude θ of an aircraft is

$$G_{e\theta} = \frac{160(s + 2.5)(s + 0.7)}{(s^2 + 5s + 40)(s^2 + 0.03s + 0.06)}$$

Estimate the rise time, settling time, and peak overshoot of the system (without doing MATLAB simulation). How accurate do you think your estimates are? Provide reasons.

Problem 12. If H is a second order transfer function without zeros, draw the region in the s-plane where the poles of H may lie so that the following specs are met: $t_r \leq 0.9$ sec, $t_s \leq 3.45$ sec, and $\zeta \geq 0.5$.

Problem 13. If a plant with transfer function $P(s) = \frac{s-2}{s^2+2s+50}$ is controlled using a standard negative feedback loop with a proportional controller, will the closed loop poles stay in the LHP for all values of the proportional gain?