Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering

Problem set 9
Issued: April 6, 2009

Problem 1. Consider the following system of coupled differential equations:

$$2\ddot{y}_1 + (\dot{y}_1 - \dot{y}_2) + 10(y_1 - y_2) = 4u$$
$$\ddot{y}_2 + 2\dot{y}_1 + 3\dot{y}_1 + 6y_2 - 4y_1 = f - u$$

find the matrix transfer function from the vector of inputs $[U(s), F(s)]^T$ to the vector of outputs $[Y_1(s), Y_2(s)]^T$.

Problem 2. Linearize the pendulum equations derived in class

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{g}{m}x_2 - \frac{g}{\ell}\sin x_1 + u \end{bmatrix}$$

around the equilibrium point $\mathbf{x}^* = [0, 0]^T$ (vertically downward position of the pendulum). Express the resulting linear system in the state space form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ and determine if this system is stable or not. (Hint: compute the eigenvalues of the matrix $\mathbf{A}$. *Do this eigenvalue calculation by hand. In the tests you will be required to compute eigenvalues of* 2×2 *matrices.*).

Problem 3. Express the following transfer function in the state space form:

$$\frac{Y(s)}{U(s)} = G(s) = \frac{1}{s^3 + 3s^2 + 5s + 56}$$

Compare the poles of $G(s)$ to the eigenvalues of the $\mathbf{A}$ matrix. (Hint: it follows that $(s^3 + 3s^2 + 5s + 56)Y(s) = U(s)$ which implies $\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 56y = u$. After this everything is straightforward.)

Problem 4. The soft landing of a lunar module descending on the moon can be modeled as shown in figure 1. Define the state variables as $x_1 = y$, $x_2 = \dot{x}_1$, $x_3 = m$ and the control as $u = \dot{m}$. Assume that g is the gravity constant on the moon. Find the state space model for the system. Is this a linear model?

Problem 5. Consider the cart-and-pendulum shown in Figure 2. (The problem of keeping the pendulum upright by moving the cart alone is an important problem that arises in many applications. For example, keeping a missile in its trajectory in the initial stages after its launch is similar to the problem of keeping the pendulum upright by applying a control force $u(t)$ on the cart.)

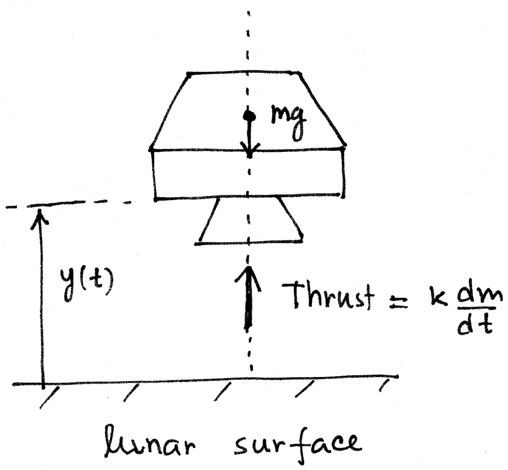

Figure 1: Lunar module landing

1. Define the state variables as $x_1 = x$, $x_2 = \dot{x}_1$, $x_3 = \theta$ and $x_4 = \dot{\theta}$, and find the state space equations for the system. Is this model linear?
2. Show that $x^* = [x_0, 0, 0, 0]$ is an equilibrium point of the system for any fixed x_0. Physically, what does this correspond to? Show that $x^* = [x_0, 0, \pi, 0]$ is also an equilibrium point.
3. Derive a linear approximation of the system by linearizing around the equilibrium point $\mathbf{x}^* = [0, 0, 0, 0]$. Express the resulting system in the standard state-space form $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$.
4. Check the stability of this linear system by computing the eigenvalues numerically for $\ell = 1\ m$, $m = 0.1\ kg$, $M = 1\ kg$ (Use `eig` command in MATLAB).

Problem 6. Consider the electromagnetic suspension system shown in Figure 3. The equation relating the position $h(t)$ of the suspended mass to the current $i(t)$ flowing through the electromagnet is:

$$m\ddot{h} = -F(i(t), h(t)) + mg + w(t), \tag{1}$$

where $w(t)$ is an external disturbance and $F(i(t), h(t))$ is the electromagnetic force:

$$F(i, h) = \frac{\mu_0 N^2 A}{4} \left(\frac{i(t)}{h(t)} \right)^2.$$

The parameters that appear in the equation above (for a Maglev train) are $m = 750kg, A = 0.021m^2, N = 324, R = 0.5\Omega, \mu_0 = 4\pi \times 10^{-7}$, and the *desired gap* is $h_0 := 0.008\ m$.

1. Determine the value of the mean current (call it i_0) so that $mg = F(i_0, h_0)$. If there were no disturbances ($w(t) \equiv 0$) and $h(\bar{t}) = h_0$, $\dot{h}(\bar{t}) = 0$ at some time $\bar{t}$, what will be the value of $h(t)$ for $t \geq \bar{t}$ if $i(t) = i_0$ for $t \geq t_0$?

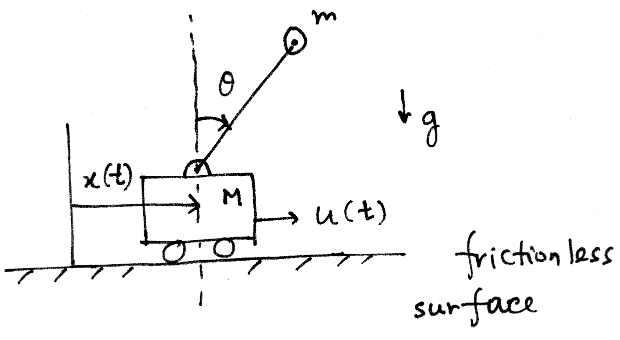

Figure 2: Cart-pendulum system. The pendulum mass m is suspended by a rigid link of length ℓ. The mass of the cart is M. Note that θ, the angular position of the pendulum, is positive in the CW direction. The position of the cart is $x(t)$, and $u(t)$ is an externally applied force on the cart. Everything is constrained to move in the plane of the paper (no three dimensional motion).

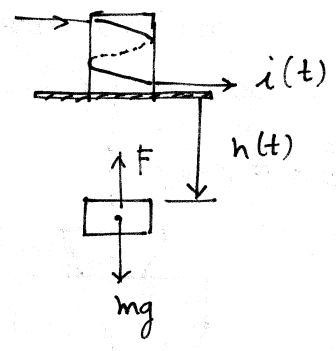

Figure 3: Electromagnetic suspension system

2. Now define $\tilde{i}(t) := i(t) - i_0$ and find a state-space representation of the system described by Eq. (1), with $\tilde{i}(t), w(t)$ as the external inputs. Is this is a linear model?
3. Determine an equilibrium point of the system.
4. Linearize this system around the equilibrium point that corresponds to $h_0(t) = h_0$ and $\dot{h}(t) = 0$. (I did not say the equilibrium point is $[h_0, 0]^T$ because the equilibrium point will depend on how you define your states.)