To design the state feedback controller ($u = -\mathbb{K}x$)

that is, to design the gain K, so that the eigenvalue of $A + \mathbb{B}\mathbb{K}$ are at $-1 \pm j$

we use Ackerman's formula

Step 1: define $q(s) = (s - \lambda_1^{(d)})(s - \lambda_2^{(d)})$

$$= [s - (-1 + j)][s - (-1 - j)]$$

$$= s^2 + 2s + 2$$

$$\Rightarrow q(A) = A^2 + 2A + 2 \cdot I_{2\times 2}$$

$$= \begin{bmatrix} -0.02 & -0.2 \\ 0.004 & 0.02 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -0.04 & -0.4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1.98 & 1.8 \\ -0.036 & 1.62 \end{bmatrix}$$

Step 2: determine

$$\mathbb{P}_c^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & -0.2 \end{bmatrix}^{-1}$$

$$= -1 \cdot \begin{bmatrix} -0.2 & -1 \\ -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 & 1 \\ 1 & 0 \end{bmatrix}$$

step 3: Apply Ackerman's formula $\mathbb{K} = [0 \quad 1] \mathbb{P}_c^{-1} q(A)$

$$\Rightarrow \mathbb{K} = [0 \quad 1] \begin{bmatrix} 0.2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1.98 & 1.8 \\ -0.036 & 1.62 \end{bmatrix} = [1.98 \quad 1.8]$$

Check Are the eigenvalues of A−BK really at $-1 \pm j$?

$$A - BK = \begin{bmatrix} 0 & 1 \\ -0.02 & -0.2 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [1.98 \ \ 1.8]$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$$

eigenvalue of A−BK can be computed now to show that they are indeed $-1 \pm j$.