Control of Mechanical engineering systems

University of Florida
Mechanical and Aerospace Engineering
Suggested exercises for midterm 1
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Chance favors the prepared mind.
- Louis Pasteur

Problem 1. Are the following statements true, false, or not possible to say without further information? Give reasons for your answers.

1. The roots of $5x^3 + 3x^2 + 4x + b = 0$, where b is a positive real number, are $-5, -4 + j, -4 + 2j$.
2. A non-linear ODE can be converted to a linear ODE by linearization without any error.
3. If a system is BIBO stable, its output will be bounded for every bounded input, but it is still possible for the output to be unbounded for certain specific initial conditions.
4. If $H(s)$ is the transfer function between the input $U(s)$ and $Y(s)$ for a system, and $u(t) = e^{-2t}$ then $y(t) = \mathcal{L}^{-1}(H(s)\frac{1}{s+2})$. ($\mathcal{L}^{-1}$ denotes inverse Laplace transform.)
5. The following transfer function describes a BIBO stable system:
$$H(s) = \frac{56s + 5}{s^5 + 2.5s^4 + 37s^2 + 678s + 3456002}.$$
6. If $y(t) = x(t)z(t)$, then $Y(s) = X(s)Y(s)$.

Problem 2. A mass (m) is suspended by a non-linear spring against gravity (Figure 1), where the spring force (f_s) is related to displacement y by $f_s = K\sqrt{y}$. (1) Find a differential equation relating the position $y(t)$ to the external force $f(t)$. (2) Find the linear differential equation relating these variables when the displacement of the mass from its equilibrium position is small.

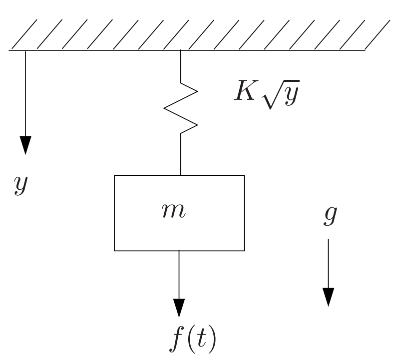

Figure 1:

Problem 3. 2.1(c) and 2.19 from the textbook

Problem 4. Let $\dddot{y}(t) + 3\ddot{y}(t) - 3\dot{y}(t) + 4y(t) = u(t) - 5d(t)$. What are the transfer functions from $U(s)$ to $Y(s)$ and from $D(s)$ to $Y(s)$? Draw block diagrams to show relationships between $U(s)$, $D(s)$ and $Y(s)$. Is the system BIBO stable?

(Express the transfer function in the form $\dfrac{b_1 s^m + b_2 s^{m-1} + \cdots + b_m s + b_{m+1}}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$ with $m < n$.)

Problem 5. Problem 2.24 from the textbook.

Problem 6.

1. Is the function $y(t) = e^{-4t+j\omega t}$ a bounded function of t for every value of ω? If so, what is the maximum value of $|y(t)|$ over all t?
2. Prove that the effect of initial conditions in the output of the following plant goes to zero as $t \to \infty$:

$$G(s) = \frac{b}{(s+1)(s-p)^2}$$

where b is a real constant and p is a complex number with $Re(p) < 0$.

Problem 7. Problem 4.11(a) from the textbook

Problem 8. The Bode plot of a transfer function $G(s)$ is shown in Figure 2.

1. If this system is subjected to an input $u(t) = 5\sin(10t)$, what is the output $y(t)$ at steady state?
2. If you are given the following two possible choices for $G(s)$, which one do you think is more likely?

$$\frac{1}{s+2} \qquad \frac{1}{s+1} \qquad \frac{8}{s^2 + 1.5s + 8}$$

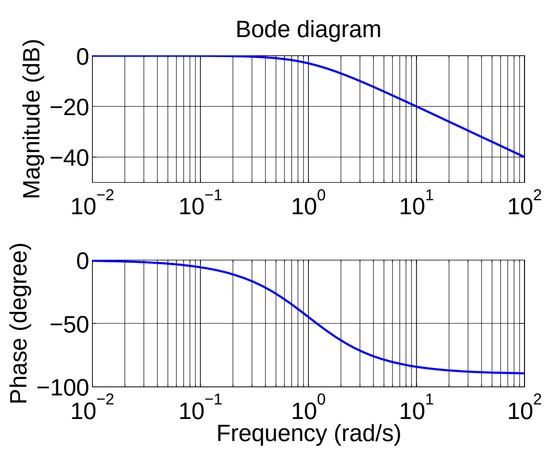

Figure 2: Bode plot of $G(s)$ for problem 8.