Control of Mechanical engineering systems

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Mechanical and Aerospace Engineering

Suggested exercises on State Space method

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Problem 1. Consider a dynamical system governed by the following set of coupled differential equations:

$$\dot{X} = aX^2 - bXY \tag{1}$$
$$\dot{Y} = Y - 2X \tag{2}$$

Is this a linear system? Verify that $(X = 0, Y = 0)$ is an equilibrium point of this system. Is it the only equilibrium point? Linearize the system around the equilibrium $(X = 0, Y = 0)$.

Problem 2. The linearized equation of motion of a high-performance helicopter are:

$$\ddot{\theta} = -\sigma_1\dot{\theta} - \alpha_1\dot{x} + n\delta + w$$
$$\ddot{x} = g\theta - \alpha_2\dot{\theta} - \sigma_2\dot{x} + g\delta,$$

where $\theta(t)$ is the pitch angle of the helicopter, $x(t)$ is its translation, the rotor thrust angle $\delta(t)$ is a control input that is used to control the pitch θ, and w is an external disturbance. $n, g, \sigma_{(\cdot)}, \alpha_{(\cdot)}$ are constants.

1. Is this a linear system?
2. Express these equations in state-space form with $\mathbf{x} = [\theta, x]^T$ as the state vector and $[w, \delta]^T$ as the input vector.

Problem 3. Consider the LTI system

$$\dot{x} = -x + y + u,$$
$$\dot{y} = 2x - 5y + v.$$

What are the poles of the transfer function from $[\mathbf{u}(s), \mathbf{v}(s)]^T$ to $[\mathbf{x}(s), \mathbf{y}(s)]^T$? Is this a stable system?

Problem 4. Consider a system given by

$$\dot{\mathbf{x}} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \end{bmatrix}\mathbf{x} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} u \text{ and } y = [1 \quad 1]\mathbf{x}$$

Check the (complete) controllability and reconstructibility of the system.

Problem 5. Design an observer to estimate the state for the system with the following transfer function from the input $U(s)$ to output $Y(s)$:

$$\frac{Y(s)}{U(s)} = \frac{5}{s^2 + 2s + 10}$$

(use y and $\dot{y}$ as the two states of the system when you convert it to state space form). In particular, choose the observer gain $\mathbf{L}$ so that the eigenvalues of $\mathbf{A} - \mathbf{LC}$ are at $-4 \pm 4j$.

Problem 6. Consider a LTI system with n states, m inputs, and p outputs:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}, \quad \mathbf{y} = \mathbf{C}\mathbf{x},$$

and an observer for its state:

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}\mathbf{u} + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}), \quad \mathbf{y} = \mathbf{C}\mathbf{x},$$

You want to perform a MATLAB simulation of the observer to check if the estimate $\hat{\mathbf{x}}$ really converges to the state $\mathbf{x}$. Express the equation for the observer in a form suitable for simulation, with state $\hat{\mathbf{x}}$ and input $[\mathbf{u}^T, \mathbf{y}^T]^T$.

Problem 7. Consider the system

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 20 & -0.5 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Design a state feedback controller for the this system so that the closed loop eigenvalues are at $-2 \pm j$.