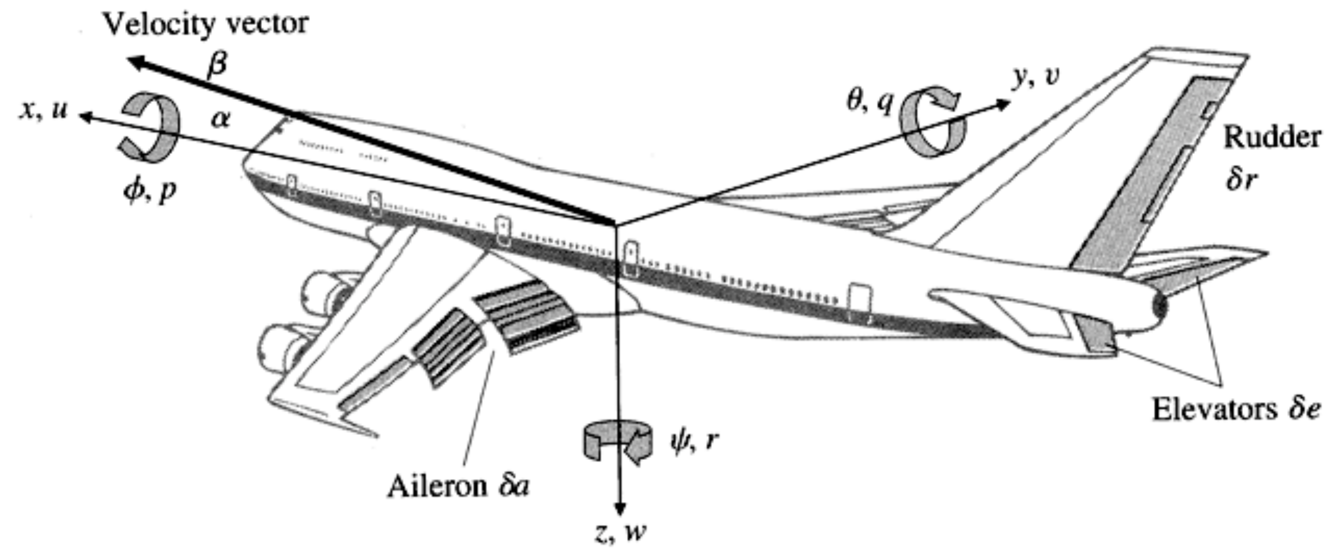
state space model example

Lateral and longitudinal control of a Boeing 747
Section 10.3 in textbook



x, y, z = position coordinates	ϕ = roll angle
u, v, w = velocity coordinates	θ = pitch angle
p = roll rate	ψ = yaw angle
q = pitch rate	β = side-slip angle
r = yaw rate	α = angle of attack

aileron and rudder primarily affect lateral motion. Although there is a small amount of coupling of lateral motion into longitudinal motion, this is usually ignored, so the equations of motion are treated as two decoupled fourth-order sets for designing the control, or **stability augmentation**, for the aircraft.

The nonlinear rigid body equations of motion in body-axis coordinates, under proper assumptions,[4] can be derived as (Bryson, 1994)

$$m(\dot{U} + qW - rV) = X - mg\sin\theta + \kappa T\cos\theta, \tag{10.16}$$

$$m(\dot{V} + rU - pW) = Y + mg\cos\theta\sin\phi,$$

$$m(\dot{W} + pV - qU) = Z + mg\cos\theta\cos\phi - \kappa T\sin\theta,$$

$$I_x\dot{p} + I_{xz}\dot{r} + (I_z - I_y)qr + I_{xz}qp = L, \tag{10.17}$$

$$I_y\dot{q} + (I_x - I_z)pr + I_{xz}(r^2 - p^2) = M,$$

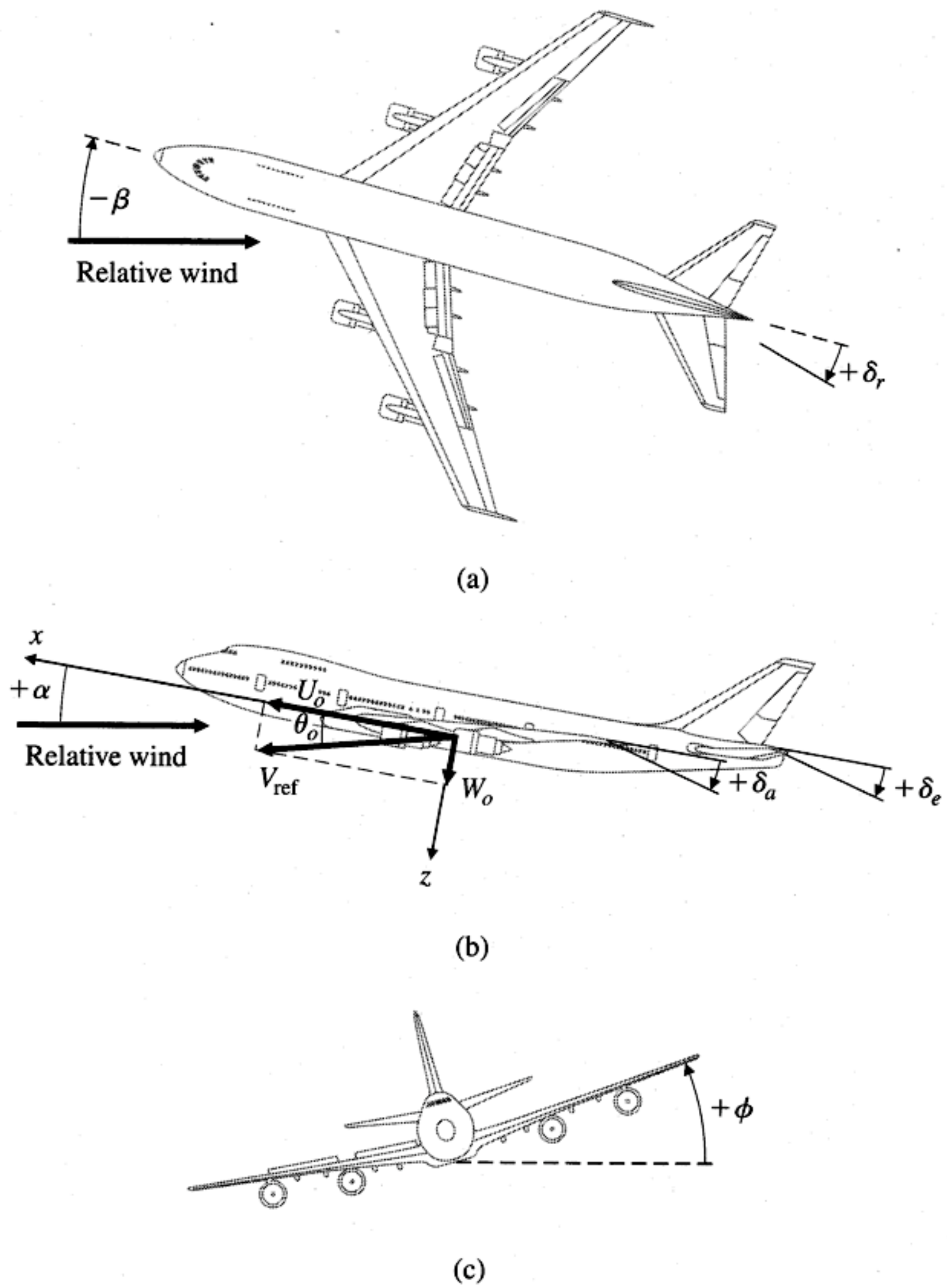
$$I_z\dot{r} + I_{xz}\dot{p} + (I_y - I_x)qp - I_{xz}qr = N,$$

where

m = mass of the aircraft,

$[U, V, W]$ = body-axis components of the velocity of the center of

10.31
state flight condition



With the assumptions (Bryson, 1994)

$$
\begin{aligned}
(v^2, w^2) &\ll u^2, \\
(\phi^2, \theta^2) &\ll 1, \\
(p^2, q^2, r^2) &\ll \frac{u^2}{b^2},
\end{aligned}
\tag{10.21}
$$

where b denotes the wingspan, many of the nonlinear terms in Eqs. (10.16) and (10.17) can be neglected. Substitution of Eq. (10.20) in the nonlinear equations of motion leads to a set of linear perturbational equations that *page number in textbook*

For linearized lateral motion, the results are

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y_v & -U_o & V_o & g_o \cos\theta_o \\ N_v & N_r & N_p & 0 \\ L_v & L_r & L_p & 0 \\ 0 & \tan\theta_o & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} Y_{\delta r} & Y_{\delta a} \\ N_{\delta r} & N_{\delta a} \\ L_{\delta r} & L_{\delta a} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta r \\ \delta a \end{bmatrix}, \quad (10.25)$$

where

β = side-slip angle, defined to be $\frac{v}{U_o}$,

r = yaw rate,

p = roll rate,

ϕ = roll angle,

$Y_{v,\delta r,\delta a}$ = partial derivative of the aerodynamic force in the y direction with respect to perturbations in β, δr, and δa,

$N_{v,r,p,\delta r,\delta a}$ = aerodynamic (yawing) moment stability derivatives,

$L_{v,r,p,\delta r,\delta a}$ = aerodynamic (rolling) moment stability derivatives,

δr = rudder deflection,

δa = aileron deflection.