Reconstructibility

The lateral perturbation equations of motion of a Boeing 747 in horizontal flight at 40,000 ft and nominal forward speed of 774 ft/sec (Mach 0.8), with the rudder chosen as the actuator, and yaw rate as the measured output:
(Heffley and Jewel, 1972)

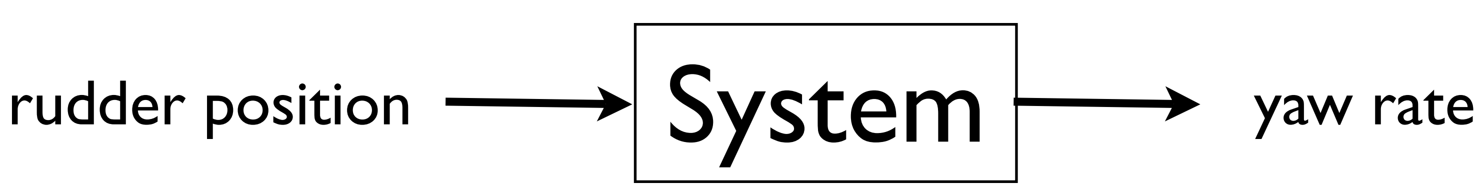
pg 747 (text)

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} -0.0558 & 0.598 & -3.05 & 0 \\ -0.9968 & -0.115 & 0.388 & 0.0805 \\ 0.0802 & -0.0318 & -0.465 & 1 \\ 0.0415 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0.00729 \\ -0.475 \\ 0.153 \\ 0 \end{bmatrix} \delta r$$

$$y = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix}$$

beta = side slip angle,
r = yaw rate
p = roll rate
phi = roll angle

delta r = rudder angle



Q: is this system reconstructible?

```
>> A = [-0.0558 -0.9968 0.0802 0.0415;0.598 -0.115 -0.0318 0;-3.05 0.388 -0.4650 0;0
0.0805 1 0]
A =
   -0.0558   -0.9968    0.0802    0.0415
    0.5980   -0.1150   -0.0318         0
   -3.0500    0.3880   -0.4650         0
         0    0.0805    1.0000         0
>> B =[0.00729; -0.475;0.153; 0]
B =
    0.0073
   -0.4750
    0.1530
         0
>>C = [0 1 0 0]
C =
     0     1     0     0
>> Po = [C; C*A; C*A^2; C*A^3]
Po =
         0    1.0000         0         0
    0.5980   -0.1150   -0.0318         0
   -0.0051   -0.5952    0.0664    0.0248
   -0.5582    0.1013    0.0125   -0.0002
>> det(Po)
ans =
  -2.4723e-04
>> rank(Pc)
ans =
     4
```

Ans: yes!

Observer design and implementation

- stable pendulum with sinusoidal excitation model :

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -g/\ell & -b/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

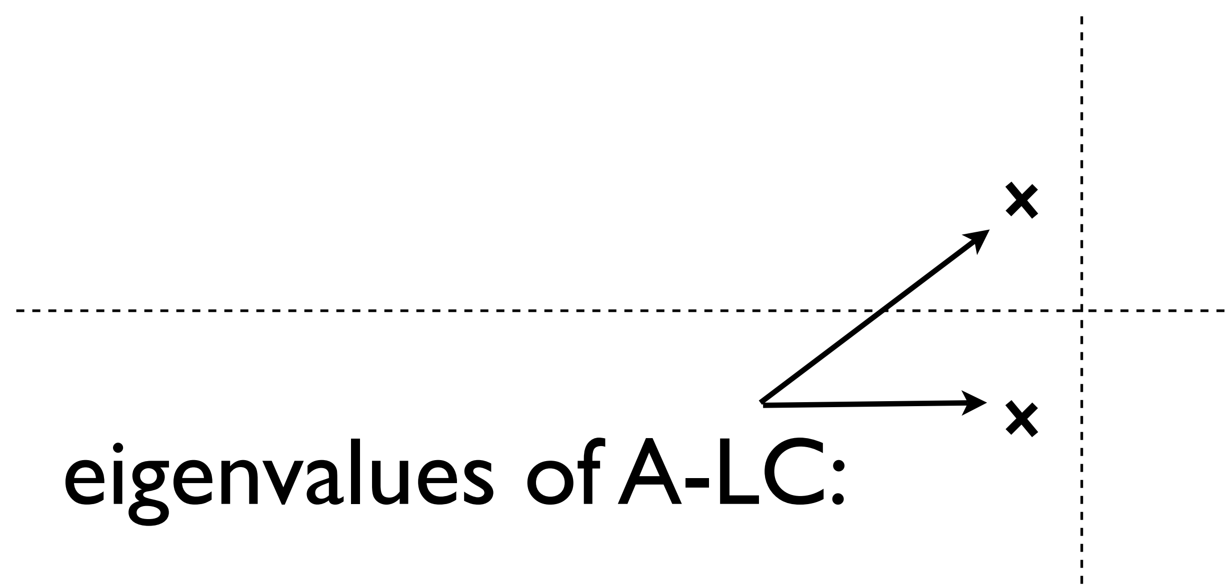
m = 0.1,
b = 0.1,
ell = 1, g
= 9.8

u(t) = 2 Hz sinusoidal signal

observer:

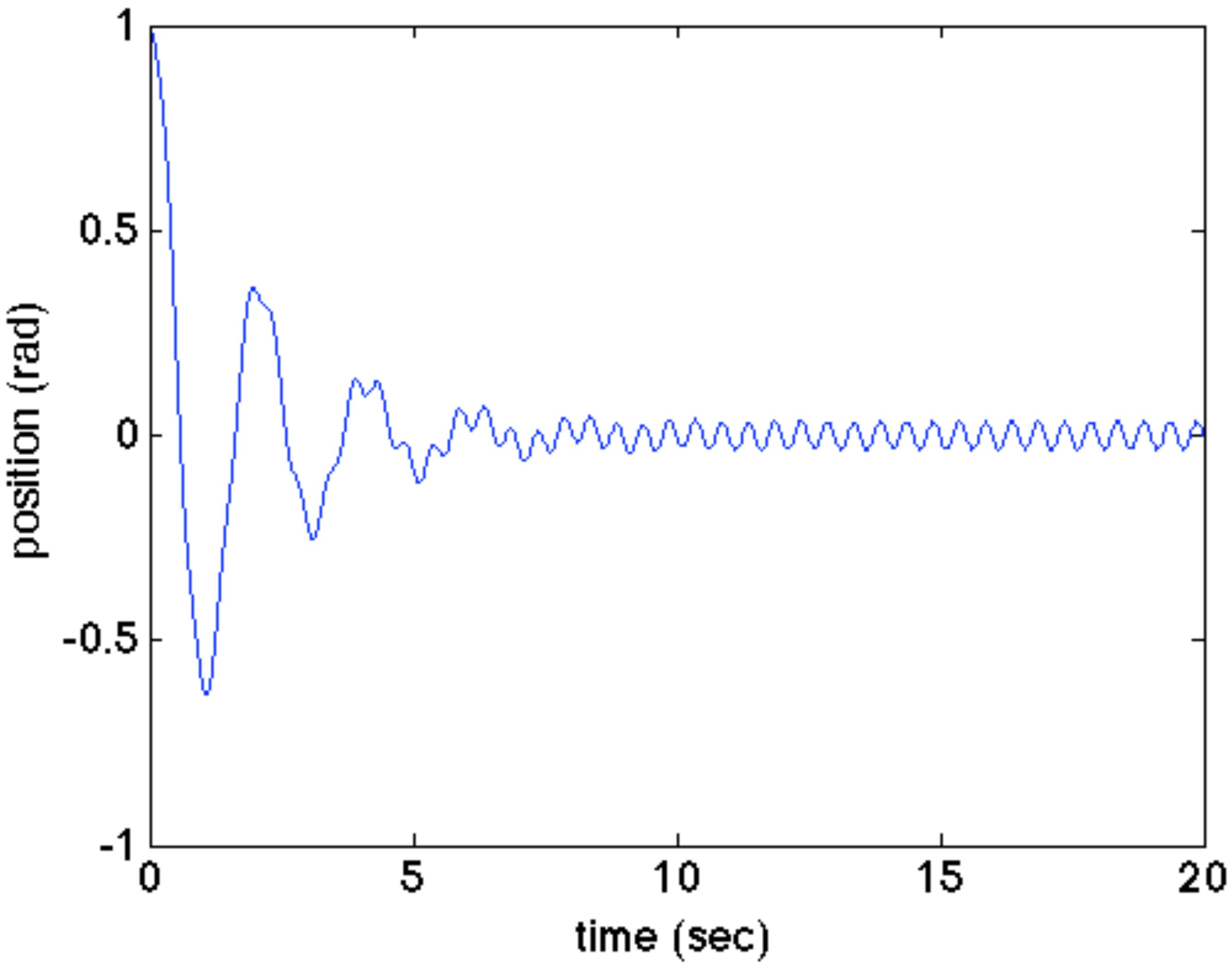
$$\begin{aligned} \dot{\hat{x}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(y - \mathbf{C}\hat{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{LC})\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}y \\ &= (\mathbf{A} - \mathbf{LC})\hat{\mathbf{x}} + \begin{bmatrix} \mathbf{B} & \mathbf{L} \end{bmatrix} \begin{bmatrix} u \\ y \end{bmatrix} \end{aligned}$$

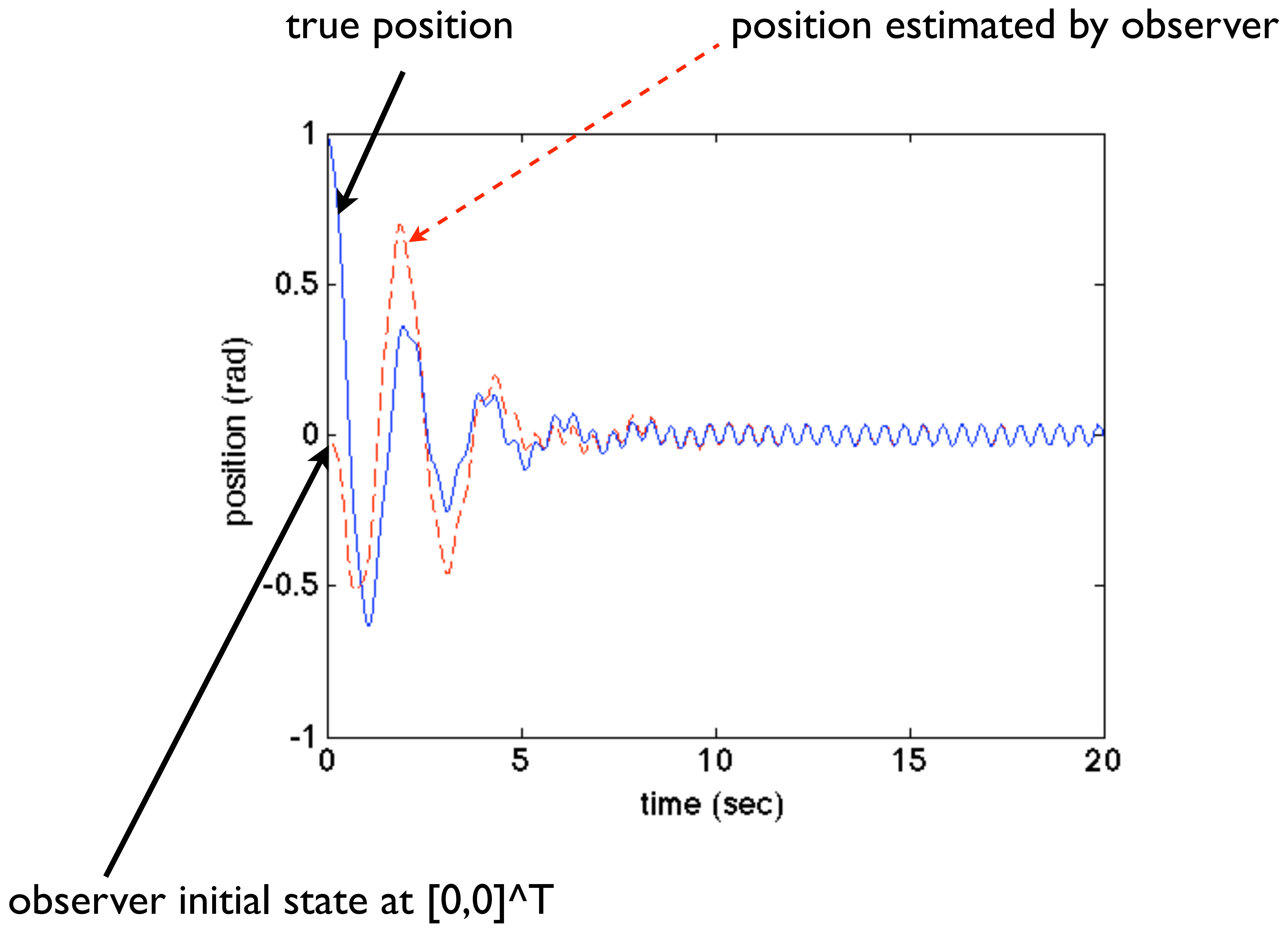
```
%% design observer:
p = [-0.5-2*j,-0.5+2*j]';
Lt = acker(A',C',p);
L = Lt'
L =
         0
   -5.5500
```

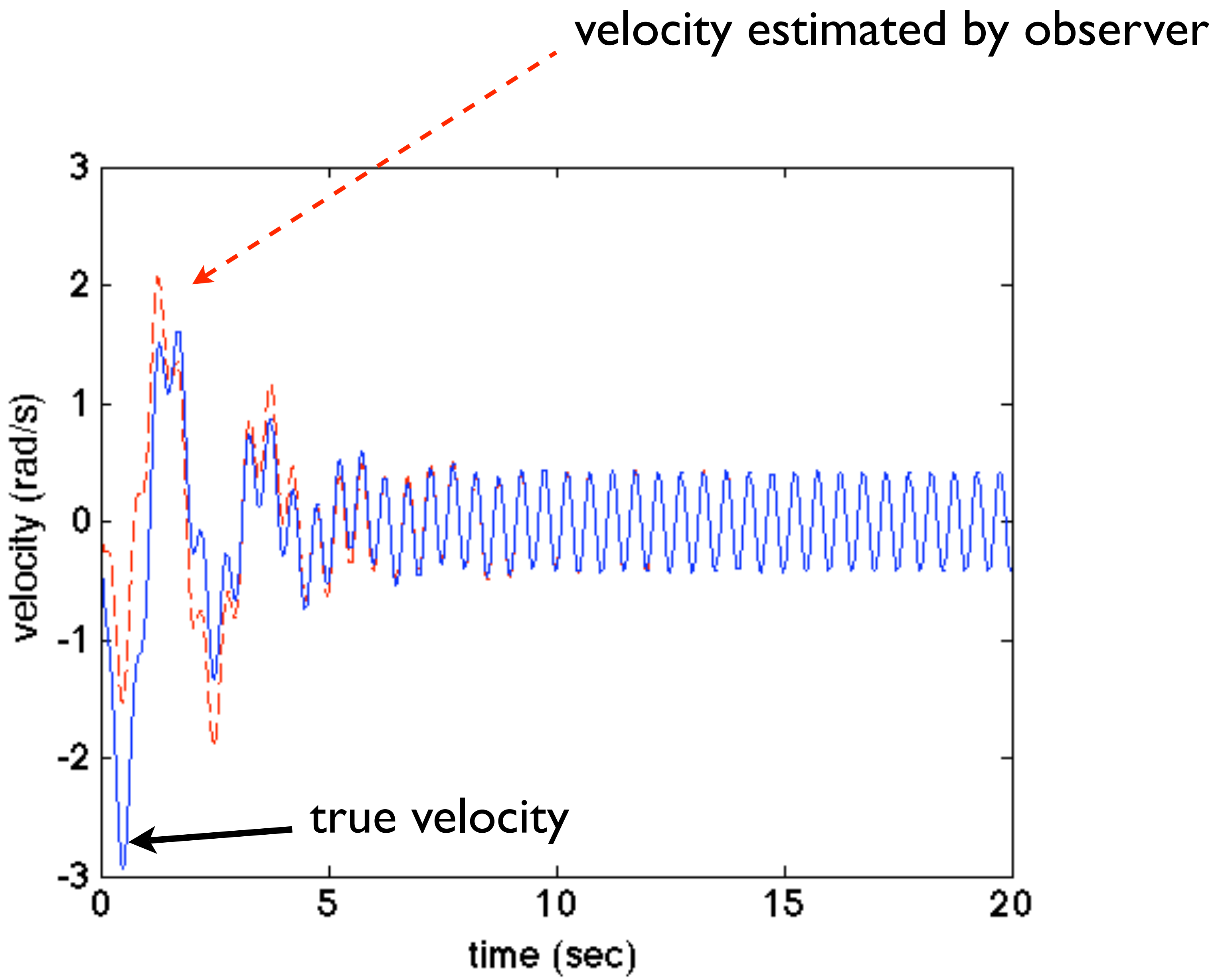


eigenvalues of A-LC:

pendulum initial condition:
pos = 1 rad, vel = -0.1 rad/s




true position
position estimated by observer
observer initial state at [0,0]^T
1
0.5
0
-0.5
-1
position (rad)
0
5
10
15
20
time (sec)

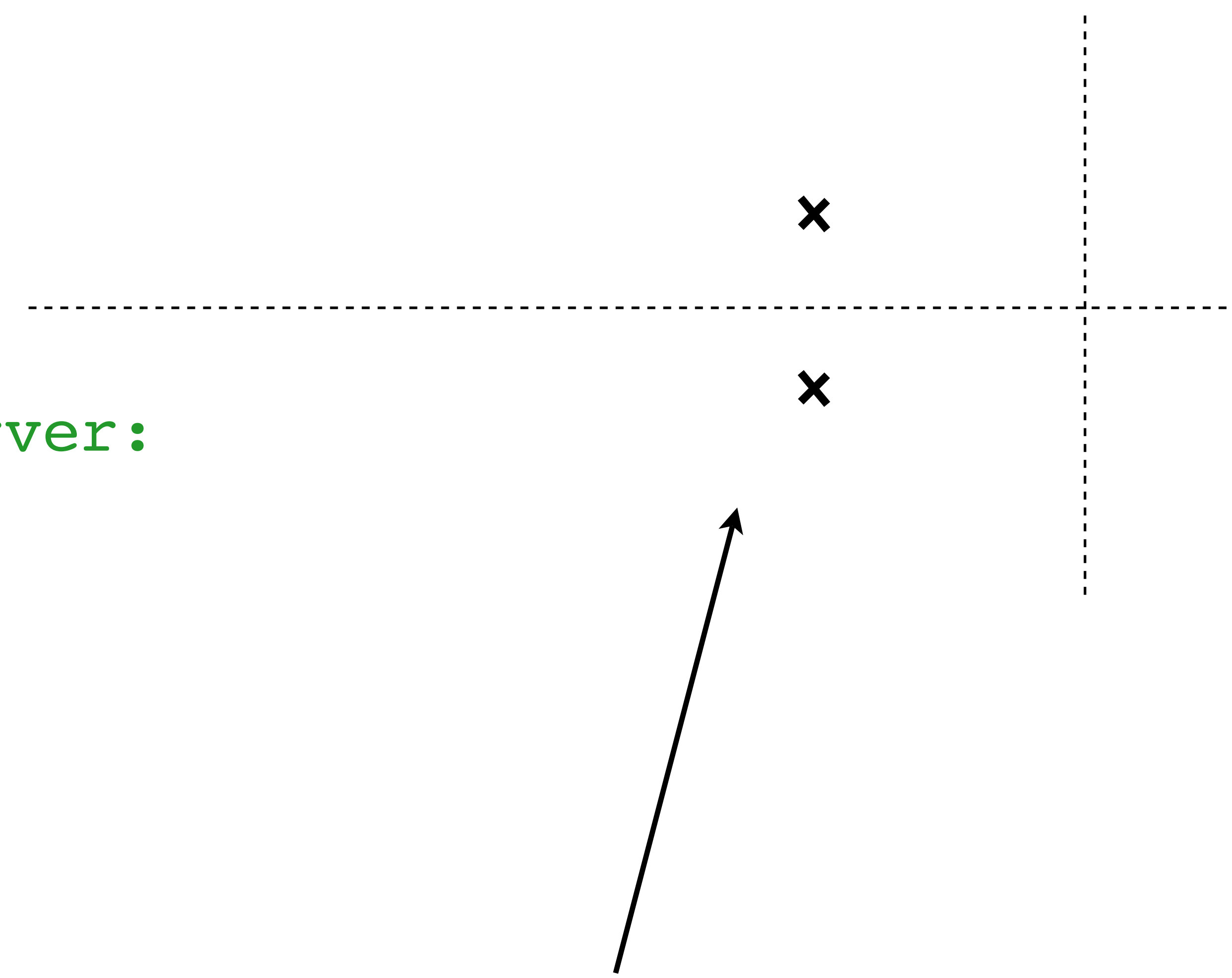

velocity estimated by observer
true velocity
velocity (rad/s)
time (sec)
3
2
1
0
-1
-2
-3
0
5
10
15
20

code

```
%%observer for a pendulum (linearized around stable equilibrium):
clear all
ell = 1;
g= 9.8;
m = 0.1;
b = 0.1;

%open loop
A = [0  1; -g/ell   -b/m];
B = [0;1];
C = [ 1  0];
D = 0;
sys = ss(A,B,C,D);

Ts = 1e-3;
time = [0:Ts:20]';
U  = 0.5*sin(2*pi*2*time)/m;

%% simulate pendulum with a certain initial condition
X0 = [1;-0.1];
[Y,tt,X] = lsim(sys,U,time,X0);
figure;
plot(time,Y);
set(gca,'fontsize',18);
xlabel('time (sec)');
ylabel(' position (rad)');


%% design observer:
p = [-0.5-2*j,-0.5+2*j]'; %desired observer eigenvalues
Lt = acker(A',C',p);
L = Lt'

%% run observer
Ao = A - L*C;
Bo = [B, L];
Co = eye(2);
Do = zeros(2);
sys_o = ss(Ao,Bo,Co,Do);

U_o = [U, Y(:)];

Xhat_init = [0,0]';
[zz,tt,Xhat] = lsim(sys_o,U_o,time,Xhat_init);

figure
plot(time,Xhat(:,1),'r--',time,Y(:),'b')
set(gca,'fontsize',18);
xlabel('time (sec)');
ylabel(' position (rad)');
```

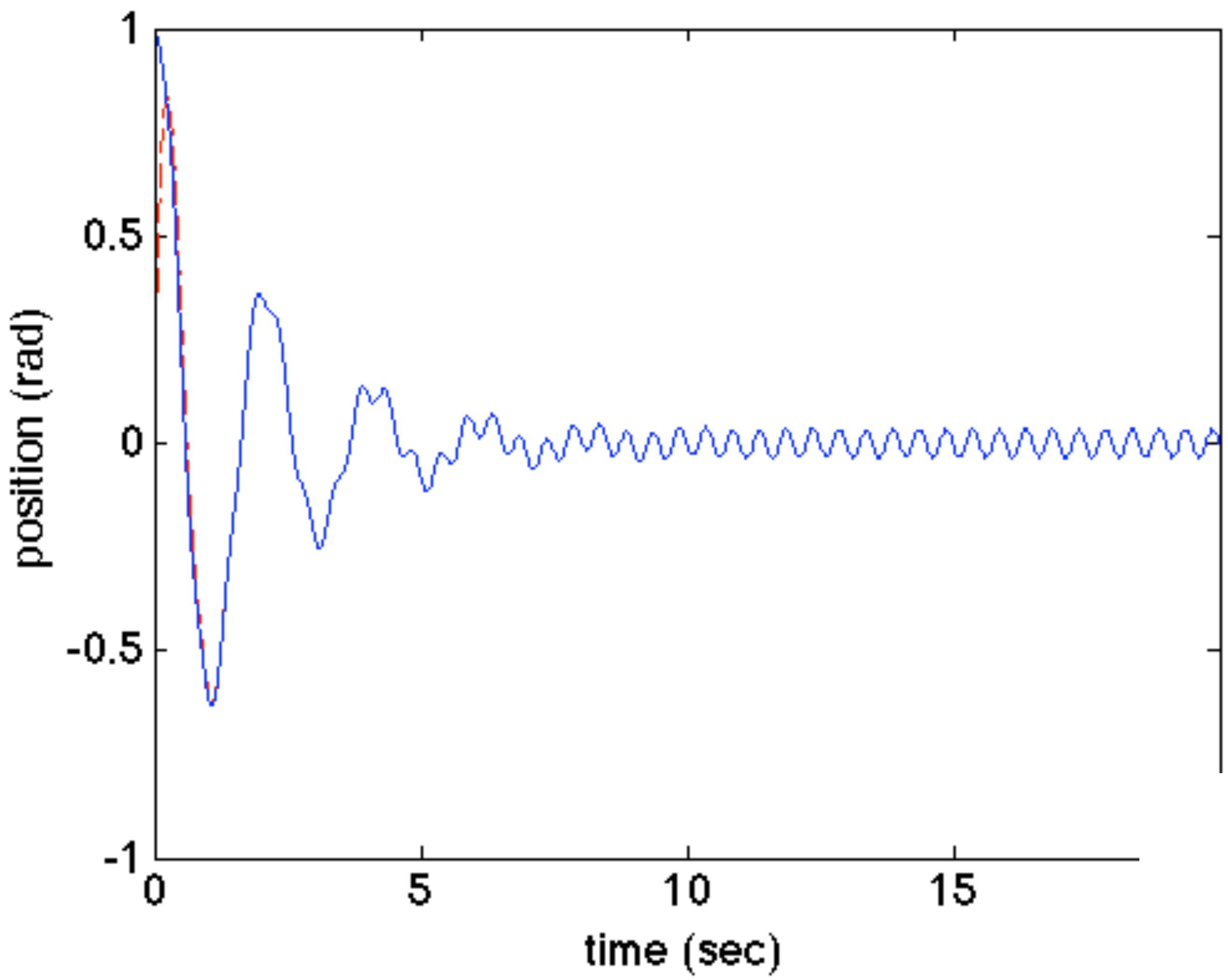
Estimation error decay rate depends on the real part of the eigenvalues of $A - LC$.

So, we can design a faster observer by choosing L so that eigenvalues of $A - LC$ are far to the left of the imaginary axis!

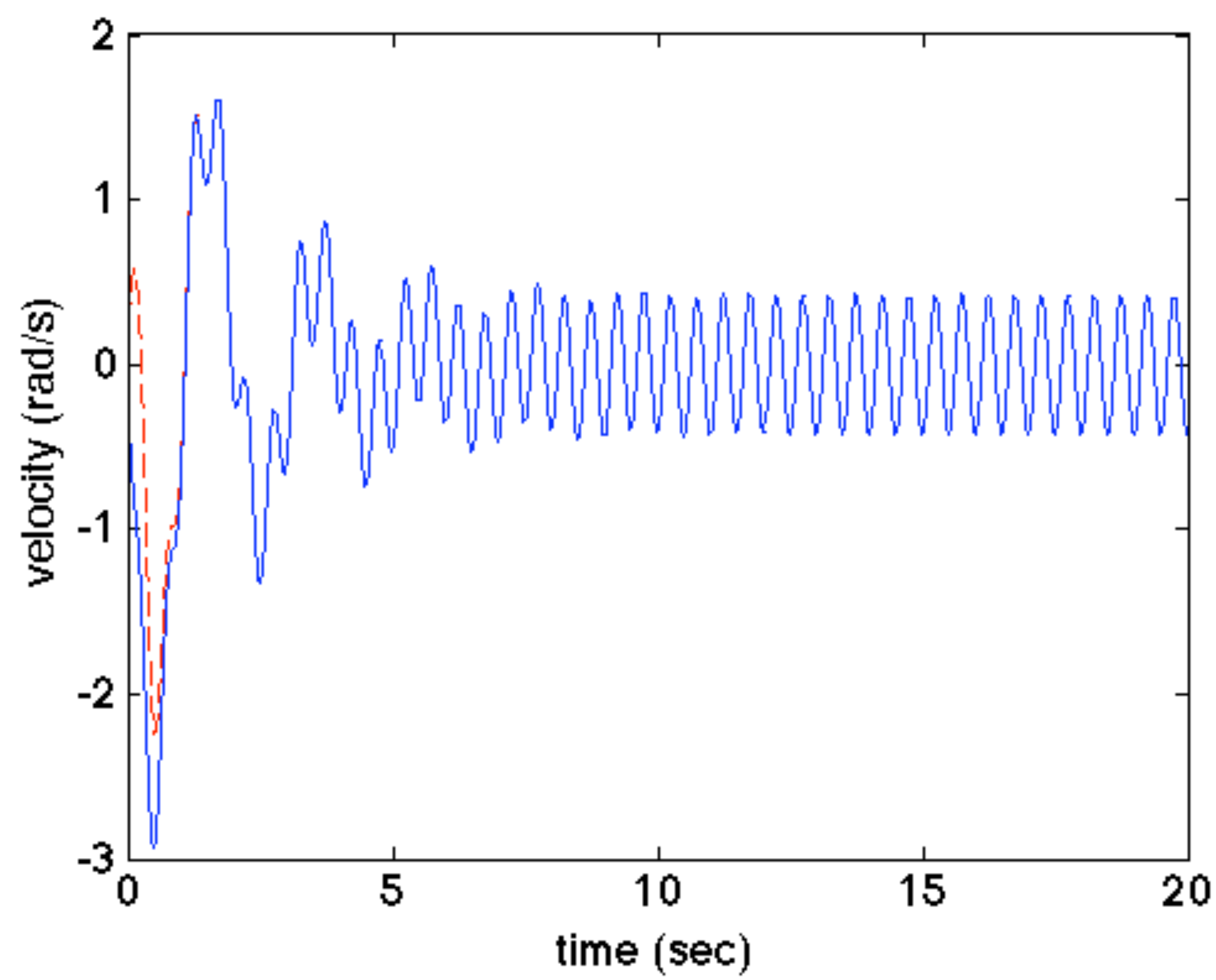


```
%% design faster observer:
p = [-5-2*j,-5+2*j]';
Lt = acker(A',C',p);
L = Lt'
L =
     9.00
    10.20
```

eigenvalues of A-LC:



1
0.5
0
-0.5
-1
0
5
10
15
position (rad)
time (sec)



2
1
0
-1
-2
-3
0
5
10
15
20
velocity (rad/s)
time (sec)