System Identification

University of Florida
Mechanical and Aerospace Engineering

HW #3

Issued: October 6, 2008
Due : in class on October 13, 2008

Problem 1. Do the two modeling exercises 2.1.2 in the lecture notes.

Problem 2. Compute the Laplace transforms of the following signals (specify the ROC's as well):

1. $y(t) = \sin(\omega t)1(t)$.
2. $y(t) = t1(t)$.

Problem 3. Your friend claims that the formula about the Laplace transform of a delayed signal $\mathcal{L}(y(t-\tau)) = e^{-s\tau}Y(s)$ must be wrong. Here's his argument:

"Consider the signal $y(t) = e^{-t}$, then the delayed signal $y(t-\tau) = e^{-(t-\tau)} = e^{\tau}e^{-t}$, which is a constant multiplying the original signal $y(t)$. Therefore, the Laplace transform of $y(t-\tau)$ is simply $e^{\tau}Y(s)$, which is not the same as $e^{-s\tau}Y(s)$."

What's wrong with this argument?

Problem 4. It is known that a linear time invariant system is BIBO stable if and only if the impulse response is absolutely integrable, that is,

$$\int_{-\infty}^{\infty} |h(t)|dt < \infty.$$

In class we proved one part of this result. Specifically, we showed that if the impulse response is absolutely integrable, then the system is BIBO stable. Prove the converse, that is, show that if the impulse response is not absolutely integrable then the system is not BIBO stable.

Problem 5. Let the Laplace transform of a signal $\{y(t)\}$ be $Y(s)$ with some ROC specified. Show that if $s_0 \triangleq \sigma_0 + j\omega_0$ belongs to the ROC, then

1. every complex number along the "vertical" line passing through s_0, i.e., the complex numbers $\sigma_0 + j\omega$ (for every real ω), also belongs to the ROC.

2. if $y(t)$ is a right-sided signal (meaning it is 0 for all $t < 0$), then every complex number lying on the "horizontal" line passing through this point and lying to the right of it, i.e., the complex numbers $\sigma + j\omega_0$ (for every real $\sigma > \sigma_0$), also belongs to the ROC.

This explains why the ROC of a Laplace transform usually looks like "$Re(s) > a$" or "$Re(s) < b$", i.e., vertical strips in the complex plane.

Problem 6. using only the formula for the Laplace transform of complex exponential signals, show that if $Y(s) = \frac{1}{s^2+\omega^2}$ with a ROC: $Re(s) > 0$, then

$$\mathcal{L}^{-1}\Big(Y(s)\Big) = \frac{1}{\omega}\sin(\omega t)1(t)$$

(Hint: use $s^2 + \omega^2 = (s - j\omega)(s + j\omega)$ and use partial fraction expansion.)

Problem 7. Use MATLAB to compute the roots of the polynomials $D(x) = 2x^3 + 15.57x^2 + 7915x + 1.184 \times 10^4$. What are the poles and zeros of

$$H_c(s) = \frac{2s+4}{2s^3 + 15.57s^2 + 7915s + 1.184 \times 10^4}?$$

Find a partial fraction expansion of $H_c(s)$ and determine the impulse response.

Problem 8. Show that following: (α is a real number)

$$\mathcal{L}(te^{-\alpha t}1(t)) = \frac{1}{(s+\alpha)^2}, \text{ ROC} : Re(s) > -\alpha$$

Problem 9. Find a partial fraction expansion of the following transfer function

$$H_c(s) = \frac{1}{(s+d)(s+a)^2},$$

where a and d are real numbers, and compute the impulse response.