System Identification

University of Florida
Mechanical and Aerospace Engineering

HW #4
Issued: Oct. 22, 2008
Due : in class on Oct. 29, 2008

Problem 1. Compute the z-transforms of the following signals ($a \in \mathbb{C}$): [4+4+2=10 pts]

$$y_1[k] = a^k 1[k], \qquad y_2[k] = -a^k 1[-k-1].$$

Sketch the two signals for $a = 1$.

Problem 2. [3+3+4=10 pts]

1. Let $y[k] \triangleq b^k 1[k]$. Write down the expression for the signal $v[k] \triangleq y[k-3]$, i.e., the one you get by delaying $y[k]$ by 3 time steps.
2. Sketch the signal $y[k]$ (for $b = 0.5$, $-3 \le k \le 5$) and the delayed signal $v[k]$.
3. Sketch the signal $x[k] \triangleq b^{k-3} 1[k]$ (for the same values of b and k as above). Are $x[k]$ and $v[k]$ the same signals?

Problem 3. [5+5=10 pt]

1. Compute a partial fraction expansion of

$$H_d(z) = \frac{z+1}{z^2 + 2z + 1}$$

2. Determine the impulse response using what you know about the z-transform of complex exponential siganals. The impulse response should be real. (Hint: use the fact that if $Z(y[k]) = Y(z)$, then the z-transform of the delayed signal $y[k-1]$ is $z^{-1}Y(z)$.)

Problem 4. [5+5=10 pt]

1. Find the unit-pulse response (after doing a partial fraction expansion) of

$$H_d(z) = \frac{z}{(z+0.2)(z-0.8)}$$

2. Write the difference equation relating the output $y[k]$ to the input $u[k]$ for this transfer function.

Problem 5. [5 pt]

Write the difference equation relating the output $y[k]$ to the input $u[k]$ for the system with transfer function:

$$H_d(z) = \sum_{k=0}^{6} k z^{-k}.$$

Problem 6. [5+10+5=20pt]

1. Show that for $y[k] \triangleq k1[k]$, $Y(z) = \frac{z}{(z+1)^2}$, ROC : $|z| > 1$.
2. Determine the impulse response of

$$H_d(z) = \frac{z}{(z+0.8)(z+0.9)^2}, \text{ ROC: } |z| > 0.9$$

3. Use MATLAB commands `impulse` to compute and plot the unit-pulse responses of the following systems:

$$H_1(z) = \frac{z}{(z+(1-\epsilon))}, \qquad H_1(z) = \frac{z}{(z+(1+\epsilon))},$$

where $\epsilon = 10^-6$. Which system is BIBO stable?

Problem 7. [5 + 5 = 10 pt]

1. Consider the continuous-time sinusoidal signal with frequency f_0 Hz: $y(t) = \sin(2\pi f_0 t)$. Let T_s be the sampling period. The sampled discrete-time signal is $y_d[k] \triangleq y(t = kT_s)$. Derive an expression for $y_d[k]$.
2. We want to express the signal $y[k] = \sin \Omega k$. Derive an expression for Ω.
3. Plot the signals $y_d[k]$ for the two frquencies $f_0 = 10$ and $f_0 = 1010$ Hz. Plot for a sampling period of $T_s = 10^{-3}$ sec., and for $k = [-100 : 100]$. (You should see that the sampled signals, for both the frequencies, are the same. This is related to the discrete-time frequency response $H_d(e^{j\Omega})$ being periodic.)

Problem 8. [5+5+5 pt]

1. Compute the discrete-time transfer function between $Y(z)$ and $U(z)$, where $y[k]$ and $u[k]$ are related by

$$y[k+2] + 2.3333y[k+1] + 4.99y[k] = u[k+1] + 5u[k].$$

2. Use MATLAB to find the poles and zeros of $H_d(z)$ determined above. Is the system BIBO stable?
3. Plot the unit-pulse response.

Problem 9 (transfer function of an unit delay)**.** [5 pt]

Suppose the input and outputs of a system are related by $y[k] = u[k-1]$. What is the discrete-time frequency response $H_d(z)$ of this system?