System Identification

University of Florida
Mechanical and Aerospace Engineering

HW #5
Issued: Nov. 3, 2008
Due : in class on Nov. 10, 2008

Problem 1. [2+5+3 +5 = 15 pt]

1. Write down the continuous time transfer function from $u(t)$ to $y(t)$, when they are related by

$$m\ddot{y} + b\dot{y} + ky = \gamma u.$$

2. Call this transfer function $H_c(s)$. Now derive the expression for the discrete time transfer function $H_d(z)$ corresponding to this continuous time transfer function with Tustin approximation (in terms of the parameters m, b, k, γ, and the sampling period T).

3. For $m = 0.1, b = 1.0053, k = 2.5266 \times 10^4$, and $\gamma = 1000$ compute the value of $H_c(j\omega)$ for a number of ω values in the range $0 \leq \omega \leq 10^5$ rad/s, and plot the magnitude and phase of $H_c(j\omega)$ so evaluated (i.e., the Bode plot). Use the MATLAB command `bode` to verify your Bode plot.

4. Compute the poles of $H_c(s)$ above for the aforementioned values of m, b, k, γ using the MATLAB `zpkdata` command. For these parameter values, numerically compute the poles of $H_d(z)$ (using MATLAB) for the following two values of T: (1) 1 msec and (2) 1 μsec. You can convert continuous time models to discrete time models in MATLAB using Tustin transformation as shown in the following example:

```
s = tf('s');
Hc = (4*s + 3)/(s^2 + 2*s +2)
T = 0.001;
Hd = c2d(Hc,T,'tustin');
```

   and then use `zpkdata(Hd,'v')` to compute the poles and zeros. Is $H_d(z)$ BIBO stable for both the sampling periods?

Problem 2. System ID [4+(5+3)*2 =20 pt]

1. Derive the difference equation corresponding to $H_d(z)$ derived above in terms of the coefficients of the numerator and denominator polynomials (the parameters γ, m, b, k are unknown).

2. Use the least squares parameter estimation technique to identify $H_d(z)$ from input output data. Use the function `F_simulate_2ndorder.p` provided in the website to drive the system with a signal of your choice and record the output. How do you make sure your are getting a "good" estimate?

3. Compute the parameters γ, m, b, k from the identified α_i's and β_i's of $H_d(z)$.

Repeat the last two tasks for two sampling periods:(1) $T = 10^{-3}$ sec and $T = 10^{-6}$ sec. Can you determine in which case you get "better" estimates?

Problem 3. [10 pt]

Let $y[k]$ be the response of the following system due to an initial condition $y[0] = y[1] = y[2] = 1$:

$$y[k+3] + 5y[k+2] + 8y[k+1] + 6y[k] = u[k]$$

with $u[k] = 0$ for all k. For an arbitrary $\epsilon > 0$, how large does K_0 have to be such that $|y[k]| < \epsilon$ for all $k > K_0$? (Hint: it'll depend on the pole of the transfer function that is closest to the unit circle, which is called the "least stable" pole.)