System Identification

University of Florida
Mechanical and Aerospace Engineering

HW #6
Issued: Nov. 14, 2008
Due : in class on Nov. 24, 2008

Problem 1. [5 pt]

The file `Problem1_data.mat` contains two variables $Omegas$ and Hd_data, where $Omegas$ is a vector of frequencies (in radians) and Hd_data is the frequency response of a discrete time transfer function at those frequencies. That is, $Hd\_data(\ell) = H_d(e^{j\Omega_\ell})$, where $\Omega_\ell = Omegas(\ell)$. The sampling period is $T = 1e - 3$. Estimate the steady state phase shift between the output and the input when the input is a 32 Hz sinusoid?

Problem 2. [5+5 = 10 pt]

Load the signal $y[k]$ (obtained by sampling $y(t)$ at $T = e - 4$) in the file `Problem2_data` into MATLAB workspace.

1. Compute the PSD of the signal (using `pwelch`) and plot the PSD as a function of frequency in Hz. Use the form [`Pxx`, `W`] = `PWELCH`(`X`, `WINDOW`, `NOVERLAP`, `NFFT`) so that the frequencies in `W` are in Radians.
2. From the estimated PSD, determine the number of sinusoidal components in $y(t)$ and their amplitudes? The choice of `NFFT` is crucial in estimating the the amplitudes accurately. Justify your choice of this parameter.

For the next two problems: the function `F_simulate_system_HW6.p` available in the website simulates a SISO linear system of unknown order. See `simulate_system_HW6.m` for an example of how to use this function. Use $T = 10^{-3}$ sec. (You can use `noise_variance` = 0 to remove measurement noise, which might be useful in debugging.)

Problem 3. [10 pt]

Use the data-correlation method to estimate the frequency response of the plant. Choose the input as a Normally distributed random sequence generated by `randn` in MATLAB. Make sure that the the standard deviation of the input (from 0) is not larger than 1. Plot the identified frequency response in the form of a Bode plot. Make sure that the abscissa is in radians (from 0 to π). Pay attention to the effect of initial conditions.

Problem 4. [20 + (5+5) + 5 pt]

1. Estimate the transfer function of the plant at M frequencies $\Omega_m \in [0, \pi), m = 1$, by sine-sweep experiments. Use unit amplitude sinusoids. The choice of frequencies Ω_m (as well as that of M) should be so as to extract the most useful information, i.e., the frequencies should be finely spaced near the peaks of $|H_d(e^{j\Omega})|$ and sparse where the frequency response is flat. (Choosing appropriate frequencies might require some iteration; you should use what you obtained in the previous Problem as a guide.) In addition, pay attention to the effect of initial conditions and measurement noise.

2. Use the MATLAB `invfreqz` function to fit a rational discrete-time transfer function to the frequency response data. Note that:
   (a) You'll need to choose appropriate orders (n and m) of the numerator and denominator polynomials. Give precise reasons for your choice.
   (b) Use an appropriate weighting on the frequency response in using `invfreqz`. Give reasons for your choice.

3. Plot the experimentally determined frequency response and overlay that on that of the fitted transfer function.

Problem 5. [7.5+7.5 = 15 pt]

1. Prove that the PSD of a real sequence $u[k]$, i.e., $P_{uu}(\Omega)$, is a real function of Ω.

2. Show that when $y[k]$ is the output sequence of a LTI plant with unit-pulse response $h[k]$ to an input sequence $u[k]$, then

$$R_{yy}[\ell] = h[\ell] R_{uy}[\ell]$$