System Identification

University of Florida
Mechanical and Aerospace Engineering

HW #1
Issued: September 8, 2008
Due : in class on Sept 15, 2008

Problem 1. Consider the following function f of the vector argument $x = [x_1, x_2]^T$:

$$f(x) \triangleq \frac{1}{2}(x_1^2 + x_2^2) - 5x_1x_2 + 4(x_1 + x_2)$$

Verify that $\frac{df}{dx} = 0 \Rightarrow [x_1, x_2] = [1, 1]$. Plot the function in MATLAB using the `surf` command as follows: (pick appropriate values of a and b)

```
[X,Y] = mesgrid([a:0.1:b],[a:0.1:b]);
f = 0.5*(X.^2 + Y.^2) - 5*X.*Y + 4*(X+Y);
surf(X,Y,f);
```

and verify that f attains neither a maximum nor a minimum at $X = [1, 1]^T$. Compute the Hessian of the function at $[1, 1]^T$.

Problem 2.

1. A square matrix A is called skew symmetric if $A^T = -A$. Given an example of a 2×2 skew symmetric matrix. Prove that if A is skew symmetric, then $x^TAx = 0$ for every vector x of compatible dimension.

2. An arbitrary square matrix A can be decomposed into a symmetric and skew symmetric part by writing it as follows

$$A = \underbrace{\frac{A + A^T}{2}}_{A_s} + \underbrace{\frac{A - A^T}{2}}_{A_k}$$

   Prove that A_s is symmetric and A_k is skew symmetric.

3. Based on the above, prove that for an arbitrary square matrix A (one that is not necessarily symmetric), one always has $x^TAx = x^TA_sx$ for every vector x.

4. A symmetric matrix is positive definite if and only if each of its eigenvalues is negative. How are you going to check positive definiteness of a (not necessarily symmetric) matrix?

Problem 3.

1. Consider the discrete time model

$$y[k+1] = ay[k] + bu[k], \quad k = 0, 1, \ldots$$

with the unknown parameters a and b. The SSE for data collected during the first N time steps, $\tilde{y}_i, \tilde{u}_i$, $i = 0, \ldots, N$, where $\tilde{y}_i = y_i + \epsilon_i$ and $\tilde{u}_i = u_i + \varepsilon_i$ are noise-corrupted measurements, is

$$SSE = \sum_{i=1}^{N} (\tilde{\mathbf{y}} - \Phi\boldsymbol{\theta})_i^2$$

where $\boldsymbol{\theta} = [a, b]^T$, and $\tilde{\mathbf{y}}$ and Φ are as defined in class. Find the least squares estimates of the parameters a and b by setting $\frac{d}{d\boldsymbol{\theta}} SSE = 0$.

2. Simulate the system in MATLAB by choosing $a = 1.01$, $b = 1$ and an initial condition $y[0]$ of your choice. In MATLAB, you can use `v * randn` to generate zero-mean random numbers with a variance of $\texttt{v}^2$. Choose `v` a small number (say, 0.2) to simulate low noise levels. Choose some value of N, say 100. Collect the simulated data and compute the least squares estimates $\hat{a}_{LS}, \hat{b}_{LS}$ for this data set.

3. Keeping everything else the same, try a few different values of N, say, $N = 100, 500, 1000, 50000, 10000$ etc. What happens to the least squares estimates? Examine the trend of the quality of fit (i.e, the normalized SSE: $\frac{SSE(\hat{\boldsymbol{\theta}}_{LS})}{\|\tilde{\mathbf{y}}\|^2}$) as a function of N.

4. The MATLAB command `A\y` can be used to directly compute the least squares solution to $A(\cdot) = y$. Use this command to find $\hat{\boldsymbol{\theta}}_{LS}$ in MATLAB.

5. For the data collected above, compute the SSE for various values of the parameters a and b and plot a surface of the SSE as a function of a, b. Does the SSE attain a minimum at $\hat{a}_{LS}, \hat{b}_{LS}$?