OPTIMAL ESTIMATION University of Florida
Mechanical and Aerospace Engineering

HW 8
Issued: October 23, 2009, Due: October 30, 2009 (in class)

Problem 1. [10 pt]

Prove that if X and Y are independent random variables,

$$f_{X|Y}(x|y) = f_X(x) \text{ and } f_{Y|X}(y|x) = f_Y(y)$$

Is there any "intuitive" explanation for these?

Problem 2. [10 pt]

Show that the conditional pdf $f_{X|Y}(x|y)$ is a "true" pdf, meaning, $f_{X|Y}(x|y) > 0$ for all x and $\int_{-\infty}^{\infty} f_{X|Y}(x|y)dx = 1$.

Problem 3. [$4 \times 5 + 5 + 5 = 30$ pt]

X and Y have a joint density given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{1}{2} & 0 \le x \le 2, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

1. Compute the marginal pdfs of X and Y and the conditional pdfs $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.
2. Are X and Y independent?
3. Compute $P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{3}{4})$

Problem 4. [$4 \times 5 + 5 + 10 + 5 = 40$ pt]

X and Y have a joint density given by

$$f_{X,Y}(x,y) = \begin{cases} 2 & 0 \le x \le y, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

1. Compute the marginal pdfs of X and Y and the conditional pdfs $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$.
2. Are X and Y independent?
3. Sketch the conditional pdfs $f_{X|Y}(x|y)$ and $f_{Y|X}(y|x)$. Mark the abscissa and ordinate, and the important values, clearly. (e.g., if you are plotting $f_X(x)$ that is uniform between 5 an 9, then the important values are 5, 9, and 1/4.)
4. Compute $P(\frac{1}{4} < X < \frac{1}{2} | Y = \frac{3}{4})$.