

OPTIMAL ESTIMATION University of Florida
Mechanical and Aerospace Engineering

HW 9

Issued: November 3, 2009, Due: November 9, 2009 (in class)

Problem 1.

[5 + 10 + 5 = 20 pt]

Consider a Poisson distributed r.v. X , whose p.m.f is given by

$$p_X(k|a) = P(X = k|a) = \frac{e^{-a} a^k}{k!}.$$

1. Find the max-likelihood estimator of the parameter a in terms of X (You already did this in an earlier HW; but be careful of the distinction between estimate and estimator).
2. Compute the CRLB (Cramér-Rao lower bound) for an unbiased estimator of a in terms of the measurement X .
3. Is the M-L estimator derived in step 1 efficient? (Hint: To compute the CRLB for an estimator involving discrete-type random variables, replace the pdf $f_X(\cdot)$ by the pmf $p_X(\cdot)$ in the formula.)

Problem 2.

[10 + 5 + 5 = 20 pt]

Let X_1, \dots, X_N be N independent, Normally distributed random variables with the same mean μ and variance σ^2 . Suppose μ is known but σ^2 is not.

1. What is the Cramér-Rao lower bound on the variance of an unbiased estimator of σ^2 (the estimator is a function of the random variables X_1, \dots, X_N)?
2. Derive the ML estimator of σ^2 (in terms of the r.v.'s X_1, \dots, X_N).
3. Is the M-L estimator of σ^2 efficient?

Problem 3.

[10 + 5 = 15 pt]

Let

$$f_{X|\theta} = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\theta)^2/2\sigma^2}$$

and the mean is modeled as a r.v. with density

$$f_{\Theta}(\theta) = \frac{\theta}{\gamma} e^{-\frac{\theta^2}{2\gamma}},$$

where γ is a known parameter. Suppose a single observation of X (call it x_0) is provided.

1. Compute the MAP estimate of the random variable Θ in terms of x_0 .
2. Compare the above with the ML estimate of the mean of X in terms of x_0 .

Problem 4.

[10 + 2.5+2.5 = 15 pt]

Suppose the conditional pdf of X given $Y = y$ is:

$$f_{X|Y}(x|y) = \begin{cases} \frac{1}{y} & 0 < x \leq y \\ 0 & x \leq 0 \text{ or } x > y \end{cases}$$

1. Compute the MMSE estimator of X , i.e., $E[X|Y]$.
2. What is the estimate when Y is measured to be 3? What is the estimate when Y is measured to be 10^5 ?