Optimal Estimation (EGM 6934, Sec 4159)

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Midterm 1
Duration: 50 minutes

There are three problems that are worth 26, 14, and 20 points, respectively. Points will be awarded for clarity and completeness of your answers.

Problem 1. The amount of time, in months, that a spacecraft functions before breaking down is modeled as an exponentially distributed continuous-type random variable X, whose p.d.f. is given by

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

1. What is the Cumulative Distribution Function (CDF) of X? [10pt]
2. Sketch the CDF. [10pt]
3. If $\lambda = \frac{1}{12.2}$, what is the probability that the spacecraft will function less than 12.2 months before breaking down? (you can leave your answer in terms of e) [6 pt]

Problem 2. 1. If X is a random variable with variance σ^2 and a is a deterministic scalar, determine the variance of aX. [6 pt]

2. If $\mathbf{X}$ is a random vector with covariance matrix Σ and A is a deterministic matrix of appropriate dimension, determine the covariance matrix of $A\mathbf{X}$. [8 pt]

Problem 3. Let X be a random variable whose pdf is given by

$$f_X(x|\theta) = \begin{cases} \frac{2}{\theta^2} x & 0 < x < \theta \\ 0 & \text{otherwise} \end{cases}$$

Suppose a single observation x_o of X is given (assume $x_o > 0$).

1. What is the likelihood function $\ell(\theta|x_o)$? [7 pt]
2. Provide a sketch of $\ell(\theta|x_o)$. [7 pt]
3. Find the max-likelihood estimator of the parameter θ given the single observation x_o. [6 pt]