Optimal Estimation (EGM 6934, Sec 4159 )

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**Midterm 2**
**Due: in class on Monday, April 7, 2008**

Points will be awarded for clarity and completeness of your answers. Write your name on all of your answer sheets before turning them in.

**Problem 1.** [5 pt]

Explain whether the following is true or not:

$$E[\frac{1}{X}] = \frac{1}{E[X]}?$$

**Problem 2.** [5 pt]

Let $Z = H\theta + \epsilon$, where $\theta$ is a deterministic parameter vector, $\epsilon$ is zero mean, and $R = Cov(\epsilon, \epsilon)$. However, $H$ is random. Consider the following proposed estimator:

$$\hat{\theta} = (H^T R^{-1} H)^{-1} H^T R^{-1} Z$$

When is $\hat{\theta}$ an unbiased estimator of $\theta$?

**Problem 3.** [10 pt]

Let $\{\omega_1, \ldots, \omega_6\}$ be the outcomes of casting a die (i.e., $\omega_i$ = number of dots that appear). Assume that $P(\{\omega_i\}) = \frac{1}{6}$ for $i = 1, \ldots, 6$, i.e., the probability of getting a number is $\frac{1}{6}$ for all the six numbers. Define two random variables $X$ and $Y$ as:

$$X(\omega) = \begin{cases} 1 & \omega \text{ is odd} \\ 0 & \omega \text{ is even} \end{cases}$$

$$Y(\omega) = \begin{cases} 0 & \omega \text{ is odd} \\ 1 & \omega \text{ is even} \end{cases}$$

Are $X$ and $Y$ independent? Provide clear arguments to support your answer.

**Problem 4.** [15 pt]

Let $X$ be a random variable that has a so-called Laplacian distribution, i.e., its pdf is given by

$$f_X(x) = \frac{1}{2\lambda} e^{-\frac{1}{\lambda}|x|}, \quad -\infty < x < \infty$$

where $\lambda$ is a deterministic parameter[1]. Let $X_1, \ldots, X_N$ be independent random variables with the same exponential distribution as $X$. You can imagine these as observations of $X$ that you have not collected yet. Design an unbiased estimator of $\lambda$ that uses $X_1, \ldots, X_N$.

[1] This distribution is important in image and speech analysis